1. Diagonalize each of the following matrices, if possible; otherwise, explain why the matrix is not diagonalizable. (Note: "Diagonalize A" means "Find a diagonal matrix D and an invertible matrix P for which $P^{-1}AP = D$." You need not compute P^{-1} if you explain how you know that P is invertible.)

   (a) $A = \begin{bmatrix} 4 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & -2 \end{bmatrix}$

2. Let $S = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ -2 \end{bmatrix} \right\}$. Find a basis for $S^{\perp}$.

3. Recall that $\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$ defines an inner product on $\mathbb{P}_2$, the space of polynomials with degree ≤ 2. Let $\boldsymbol{\tau} \in \mathbb{P}_2$ be the polynomial $\boldsymbol{\tau}(t) = t$. Find the unit vector $\hat{\boldsymbol{\tau}}$ in the direction of $\boldsymbol{\tau}$.

4. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 3 \\ 8 \\ 4 \end{bmatrix}$.

   (a) Show that $\mathcal{U} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$.

   (b) Find $[\mathbf{x}]_{\mathcal{U}}$, the $\mathcal{U}$-coordinate vector of $\mathbf{x}$.

5. Let U and V be $n \times n$ orthogonal matrices. Show that UV is an orthogonal matrix.

6. Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\{\mathbf{w}_1, \ldots, \mathbf{w}_p\}$, and let $\{\mathbf{v}_1, \ldots, \mathbf{v}_q\}$ be an orthogonal basis for $W^{\perp}$.

   (a) Explain why $\{\mathbf{w}_1, \ldots, \mathbf{w}_p, \mathbf{v}_1, \ldots, \mathbf{v}_q\}$ is an orthogonal set.

   (b) Explain why $\{\mathbf{w}_1, \ldots, \mathbf{w}_p, \mathbf{v}_1, \ldots, \mathbf{v}_q\}$ spans $\mathbb{R}^n$.

   (c) Show that $\dim W + \dim W^{\perp} = n$.