Math 18: Lectures ABC  Midterm Exam 1 v. 1  Name: ____________________
January 31, 2018  (Total Points: 30)  PID: ________________

Instructions
1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on every page.
3. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
4. You may use one handwritten page of notes, but no books or other assistance during this exam.
5. Read each question carefully and answer each question completely.
6. Write your solutions clearly in the spaces provided; only work on this exam booklet will be graded.
7. Show all of your work. No credit will be given for unsupported answers, even if correct.

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(8 points) 1. Let $A$ be any $10 \times 8$ matrix. For each statement about $A$, circle $T$ if it is always True; circle $F$ if it is ever False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

( T  F ) The matrix equation $Ax = b$ is consistent for every $b \in \mathbb{R}^{10}$.

( T  F ) The homogeneous system $Ax = 0$ is consistent.

( T  F ) The columns of $A$ are linearly independent.

( T  F ) $A(2x) = 2Ax$ for all vectors $x \in \mathbb{R}^{8}$.
2. (9 points)

The matrix \( A = \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -1 & -1 & 3 \end{bmatrix} \) has reduced row-echelon form
\[
\begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\]

(a) Describe the general solution to the vector equation
\[
x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}.
\]

(b) Do the columns of \( A \) span \( \mathbb{R}^3 \)? Briefly explain your answer.

(c) Are the first and fourth columns of \( A \) linearly independent? Briefly explain your answer.
(6 points) 3. Consider the following system of equations.

\[
\begin{align*}
x_1 - x_2 &= h \\
x_1 + hx_2 &= 4h \\
2x_1 - 2x_2 &= 4
\end{align*}
\]

(a) Find the unique value of \( h \) for which the system is consistent, and determine the solution set in that case.

(b) Consider the corresponding \textit{homogeneous} system with the same coefficient matrix. Find the unique value of \( h \) for which this system has infinitely many solutions, and describe the solution set in parametric form in this case.
4. Let $u$, $v$, $w$ be three vectors in $\mathbb{R}^4$, with the following properties: \{u, v\} are linearly independent, and $w$ is not in the span generated by $u$ and $v$.

(a) Explain why $\{u, v, w\}$ are, in fact, linearly independent. [Hint: It might be easier to explain why it is impossible for the vectors to be linearly dependent.]

(b) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be a linear transformation satisfying

$$T(u) + T(v) = T(2u + w).$$

Is $T$ one-to-one? Justify your answer.