

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 20A.
Final Examination
December 9, 2008

Turn off and put away your cell phone.

No calculators or any other electronic devices are allowed during this exam.

You may use one page of notes, but no books or other assistance during this exam.

Read each question carefully, and answer each question completely.

Show all of your work; no credit will be given for unsupported answers.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	9	
9	6	
Σ	57	

1. Find the following limits:

(a) (2 points) $\lim_{x \rightarrow 0} \frac{3^x - 1}{x}$

(b) (2 points) $\lim_{x \rightarrow 0^+} x \log(x),$

(c) (2 points) $\lim_{x \rightarrow 0^+} x^{3x}.$

2. Compute the following integrals:

(a) (3 points) $\int_0^4 \sqrt{x} \, dx$

(b) (3 points) $\int_0^4 \sqrt{16 - x^2} \, dx$ (Hint: Use geometry and interpret as an area.)

3. (6 points) Find the point(s) on the ellipse given by $x^2 + 3y^2 = 1$ where the tangent line has slope 1.

4. Let $f(x) = \sqrt{\frac{x+2}{x-2}}$.

(a) (3 points) Find the domain and range of f .

(b) (3 points) Find f^{-1} , the inverse of f . Be sure to state the domain and range of f^{-1} .

5. The function $f(x) = Ax^3 + 2x^2 - 20$ has an extreme point (that is, a local maximum or local minimum) at $x = -\frac{1}{3}$.

(a) (2 points) Find A .

(b) (2 points) Is the extreme point at $x = -\frac{1}{3}$ a local maximum or local minimum?

(c) (2 points) Find the x -coordinate of any inflection point(s) the function may have.

6. (6 points) A plot of ground is to be enclosed by a rectangular fence. Fencing for three sides costs \$8 per foot of length. Fencing for the the fourth side costs \$24 per foot of length. Use the techniques of calculus to find the maximum area that can be enclosed for \$8000. (Be sure to give evidence that this really is the maximum.)

7. (6 points) A 17-foot ladder is leaning against a wall. The bottom of the ladder slides away from the wall at 2 feet per second. How fast is the area of the triangle formed by the ladder, wall, and floor changing when the bottom of the ladder is 8 feet from the wall? (Note: $8^2 = 64$, $15^2 = 225$, and $17^2 = 289$; thus, $8^2 + 15^2 = 17^2$.)

8. Find $\frac{dy}{dx}$ for each of the following functions.

(a) (3 points) $y = \cos\left(\frac{x}{x+2}\right)$

(b) (3 points) $y = \sqrt{\sin(5x)} + \pi^2$

(c) (3 points) $y = x\sqrt{x + \sqrt{x}}$

9. The Fresnel Sine Function $S(x)$ is defined by

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt.$$

(a) (3 points) Find all *positive* critical points of the Fresnel Sine Function $S(x)$.

(b) (3 points) Determine which critical points are local maxima and which critical points are local minima.