Math 20C **Homework 5** **Due November 8, 2019**

Note: *The score you earn will be based on the correctness of your solutions. A "right answer" will earn no credit without a correct solution to support it.*

(6 points) 1. Show that the following functions satisfy the one-dimensional wave equation $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 f}{\partial t^2}$.

(a) $f(x,t) = \sin(x - ct)$

(b) $f(x,t) = \sin(x)\sin(ct)$

(6 points) 2. A function $u(x,y)$ with continuous second partial derivatives satisfying Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is called a ***harmonic function***. Show that the function $u(x,y) = x^3 - 3xy^2$ is harmonic.

(6 points) 3. Find and classify all critical points of

$$f(x,y) = \frac{1}{3}x^3 + \frac{1}{3}y^3 - \frac{1}{2}x^2 - \frac{5}{2}y^2 + 6y + 10.$$

(6 points) 4. Let $f(x,y) = 1 + xy - 2x + y$ and let D be the triangular region in $\mathbb{R}^2$ with vertices $(-2,1)$, $(-2,5)$, and $(2,1)$. Find the absolute maximum and absolute minimum values of f on D. Give all points where these extreme values occur.

(6 points) 5. Consider the function $f(x,y) = x^2+xy+y^2$ defined on the unit disc $D = \{(x,y) \mid x^2 + y^2 \leq 1\}$.

(a) Use the method of Lagrange multipliers to locate the maximum and minimum points for f on the unit circle.

(b) Use the result of part (a) to determine the the absolute maximum and minimum values for f on D.

(6 points) 6. (a) Find three numbers whose product is 27 and whose sum is minimal.

(b) Find three numbers whose sum is 27 and whose product is maximal.