Final Exam Review Problems

1. The temperature at the point (x, y) is given by a function T satisfying $T_x(5,1) = 3$ and $T_y(5,1) = -2$. A bug crawling along the plane is located at the point $(t^3 - t - 1, \sqrt{2t-3})$ at time t. Compute the rate of change of the temperature along the bug's path at the time $t = 2$.
2. Find the distance between the planes $6x + 3y - 2z = 0$ and $6x + 3y - 2z = 8$.
3. Find (and classify) the critical values of the function $f(x, y) = x^3 + xy^2 - 13x - 4y$.
4. If a box of girth 100 inches is sitting on the floor, compute the largest possible *exposed* surface area. [**Note:** the "girth" of a box is the sum of its length, width, and height.]
5. Let $A = (1, 1, 1)$, $B = (2, 8, 1)$, $C = (5, 4, -4)$.
   - **(a)** Find the angle between the lines $\overline{AB}$ and $\overline{AC}$.
   - **(b)** Compute the area of triangle $\triangle ABC$.
   - **(c)** Find the equation of the plane through A, B, and C.
   - **(d)** The line through the point $(-2, 5, p)$ is perpendicular to the plane from (c) and passes through one of the points $\{A, B, C\}$. Compute p, and determine which of the three points the line contains.
6. Early yesterday morning I was hiking on the surface $z = 3x^2 + 5xy + 8y$, but my compass wasn't working. I wandered around randomly, and the only thing I know for sure was that I was heading away from the sun (which was in the East), because it was so bright. At one point I came to a sign telling me I was at the point $(-1, 2, 9)$.
   - **(a)** If I want to maintain my exact elevation, in which direction should I walk?
   - **(b)** After having an energy drink or three, I felt ready to start climbing. If I want to walk up the hill at the steepest possible angle, what direction should I head in, and at what angle will I be climbing?
   - **(c)** After the caffeine crash, I decided to head downhill. If I'm starting at the same point, what's the steepest slope of descent I could head in, and what unit direction should I take?
7. A cone is constructed with diameter 3 cm and height 2 cm, but the variances on the measurements are both 1 mm. Compute the variance for the surface area of the cone. [**Note:** the area of the slanted side of a cone is $\pi r s$, where s is the slant height.]
8. Find the mass of a lamina in the plane bounded by the lines $y = 0$ and $y = \sin x$, where $\frac{\pi}{4} < x < \frac{3\pi}{4}$ and the density function is $\rho(x, y) = x + 2$.
9. Evaluate the following integrals:
   - **(a)** $\iint_R \frac{e^{xy}}{y}\, dA$, where R is the region bounded by the curves $x = 1$, $y = 1$, and $x^3 y = 1$.
   - **(b)** $\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} xz\, dz\, dx\, dy$
   - **(c)** $\iiint_R z\, dV$, where R is the region $4 \leq x^2 + y^2 + z^2 \leq 9$ sitting above the plane $z = 0$.
   - **(d)** $\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy$
10. Find the volume of the region between the cone $z = 3r$ and the sphere $\rho = \sqrt{10}$. (You have to choose which coordinate system to work with.)
11. The location of a particle at time t is $(\sin(2t), e^{-t}, \ln(4t + 1))$.
    - (a) Find the velocity, acceleration, and speed at time $t = 0$.
    - (b) Set up, but do not evaluate, an integral for computing the total distance traveled by the particle over the interval $0 \leq t \leq 5$.
    - (c) Show that the the speed of the particle is never greater than 3 for $t \geq 0$.
12. Compute the maximum value of $x^3 y^2 z$ given $\sqrt{x} + \sqrt{y} + \sqrt{z} = 4$.
13. Find the point on the line $4y - x = 6$ closest to the circle $(x - 11)^2 + y^2 = 1$.