Midterm Review! The harder problems are marked with an asterisk (*) and supplied with hints.

Review Problems

1. Compute the angles between the following vectors:
- **(a)** $\langle 1, 2\rangle$ and $\langle -8, 4\rangle$
- **(b)** $\langle 5, -3, 4\rangle$ and $\langle 0, 7, -1\rangle$
- **(c)** $\langle 1, 1, 0, -1, 0\rangle$ and $\langle 3, 2, -1, -7, 1\rangle$

2. Find the area of the triangle with corners $(0, 2, 4)$, $(-1, -1, 6)$, and $(1, 3, 1)$.

3. Find all possible values for x if the angle between $\langle 3, 4\rangle$ and $\langle 1, x\rangle$ is
- **(a)** 0
- **(b)** $\pi/2$
- **(c)** π
- ***(d)** $2\pi/3$ (don't bother simplifying if you get something ugly-ish)

4. Compute the sine of the angle between the vectors $\langle 2, 1, 2\rangle$ and $\langle 0, -1, 3\rangle$.

***5.** Find all x such that the vector $\langle 3, x, x^2\rangle$ lies on the plane which
- is perpendicular to the plane through the origin containing the vectors $\langle 3, -1, 3\rangle$ and $\langle -1, -7, 11\rangle$; and
- bisects the angle between those two vectors.

[**Hint:** Try drawing a picture using "nicer" vectors to get an idea of what the plane looks like in relation to the vectors.]

6. Find the equation of each of the planes described below:
- **(a)** the plane parallel to $x + 7y - 9z = 5$ and passing through $(2, 2, 2)$.
- **(b)** the plane perpendicular to the line of intersection of the planes $x + 4y - z = 6$ and $2x - y - 3z = 1$, and passing through $(4, 0, 4)$.
- **(c)** the plane parallel to the lines $\langle 3t + 5, t, 8 - t\rangle$ and $\langle 1 - t, 2 - t, t + 4\rangle$ and passing through $(5, 5, 1)$.
- **(d)** the plane containing the line $\langle 7t+7, 5t-5, 2t+2\rangle$ and perpendicular to the line $\langle t+\pi, 3t-e, 10^{100}-2t\rangle$.
- **(e)** the plane containing all points that are equidistant from $(3, -7, 8)$ and $(-1, 1, 0)$.

7. Find the velocity, speed, and acceleration of the following vector-valued functions:
- **(a)** $\vec{x}(t) = \langle t^3 + 9, 7\ln t, e^{-2t} - 2\rangle$
- **(b)** $\vec{x}(t) = \langle \sin 3t, -\cos 3t, 4t\rangle$
- **(c)** $\vec{x}(t) = \langle 3t + 6, 11 - 4t, 12t + 9\rangle$

8. Find the arclength of the following curves:
- **(a)** $\langle \sin 5t, -12t, -\cos 5t\rangle$, over the interval $0 \le t \le 2$
- **(b)** $\langle 3t^2 + 8, 2t^3 - 7, 5 - 3t\rangle$, over the interval $-1 \le t \le 1$
- **(c)** $\langle 3e^t - 8e^{-t} + 13, 4e^t + 5 + 6e^{-t}, 23 - 10t\rangle$, over the interval $0 \le t \le 1$

9. Find the minimum speed of the curve $\langle t^3 - 15t, 3t^2 - 5, 3t - 2t^3\rangle$.

***10.** Four of the eight corners of a cube are $(-1, -1, -1)$, $(1, 2, 5)$, $(2, 7, -6)$, and $(4, 10, 0)$. Find the coordinates for the center of the cube.

[**Hint:** Look at the distances between the points.]

***11.** The vertices of a triangle in the 3rd dimension are at points A, B, and C. Let $\vec{u}$ be the vector from A to B, $\vec{v}$ the vector from B to C, and $\vec{w}$ the vector from C back to A. If $|\vec{u}| = 5$ and $\vec{u} \cdot \vec{v} = 23$, compute $\vec{u} \cdot \vec{w}$.

[**Hint:** What can we say about $\vec{u} + \vec{v} + \vec{w}$?]

***12.** A cube of side length 1 in $\mathbb{R}^3$ is sitting in the first octant with one corner at the origin. The plane $x + y + z = \frac{3}{2}$ cuts out a cross section of the cube in the shape of a regular hexagon (regular means equal side lengths and angles). Find its area.

[**Hint:** Draw a picture of the region the plane cuts from the first octant.]