Midterm Review! The harder problems are marked with an asterisk and supplied with hints (*).

Review Problems

1. Use a linear approximation to estimate the following quantities:
  (a) $x^3 + y^3 + z^3$ at $(1.02, -.99, 2.03)$; base point at $(1, -1, 2)$
  (b) $(\arctan x)(\arctan y)$ at $(-1.04, .97)$; base point at $(-1, 1)$

2. Find the distance between the point $(1, 2, 3)$ and the plane $3x - 7y + 4z = 0$.

*__3.__ A cylindrical can has its base sitting in the xy-plane. The bottom circle passes through the origin and the opposite point on the top circle lies in the first octant on the plane $x + 3y + 5z = 9$. Compute the largest possible volume for the can.
[**Hint:** If that top point is (x, y, z), what is the radius of the base in terms of x and y?]

4. Find all values of k such that the function $u(x, y) = x^3 + kxy^2$ satisfies Laplace's equation $u_{xx} + u_{yy} = 0$.

5. Identify the critical values of the following functions:
  (a) $f(x, y) = x^3 + 2y^3 + x^2y - 3x^2 - 3y^2$
  (b) $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$
  (c) $f(x, y) = e^x(x^2 + y^2)$

6. A particle on the surface $f(x, y) = x^2y^2 - 3xy + x + y$ at $(1, 2)$ heads in the unit direction $\vec{u}$. If the directional derivative is 0, compute all possible vectors $\vec{u}$.

7. Compute the gradient of f at the following points:
  (a) $f(x, y) = xy + x^2y^2$ at $(-1, 3)$
  (b) $f(x, y, z) = x\sin(yz)$ at $(2, \pi, \frac{1}{3})$
  (c) $f(x, y, z) = e^{xy+3z}$ at $(1, 3, -1)$
  (d) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(2, -3, 6)$

8. Find the directional derivative of f at the point p in the direction $\vec{u}$:
  (a) $f(x, y) = 2x^2 - 4xy + y$, $p = (2, 3)$, $\vec{u} = \langle -4, 3\rangle$
  (b) $f(x, y, z) = xy - xz + yz$, $p = (2, 3, 5)$, $\vec{u} = \langle 1, 8, -4\rangle$
  (c) $f(x, y, z) = e^{xy} + e^{xz} + yz$, $p = (0, 2, -1)$, $\vec{u} = \langle 2, -1, 2\rangle$

*__9.__ A triangle has its vertices on the circle $x^2 + y^2 = 9$. Find the largest possible area for the triangle.
[**Hint:** The area of a triangle with side-angle-side of a, θ, b is $\frac{1}{2}ab\sin\theta$.]

10. Find the direction of maximum increase of the function $f(x, y) = \sqrt{x^2 + 8xy + y^2}$ at the point $(3, 4)$.

11. The wind speed at the point (x, y) is given by $S(x, y) = (x - y)(x + 2y - 3)$. A bug crawls along the path $x(t) = 1 + \sqrt{t}$, $y(t) = 1 - \sqrt{t} + t$. How fast is the wind speed changing on the bug's path at $t = 4$?

12. Compute each of the following integrals:
  (a) $\iint 2xy + 2x + y\, dA$, where R is the rectangle $[1, 2] \times [1, 3]$
  (b) $\iint 3y - 4x\, dA$, where R is the region between the lines $y = 2$, $x = -1$, $y = 5$, and $x = 3$
  (c) $\iint \frac{x}{y} - \frac{y}{x}\, dA$, where R is the square with opposite vertices $(1, 1)$ and $(10, 10)$
  (d) $\iint x\cos(xy)\, dA$, where R is the a rectangle with opposite vertices $(0, 0)$ and $(3, \pi)$

*__13.__ The ellipsoid $2x^2 + y^2 + 3z^2 = 9$ intersects the plane $3x + 2y + 6z = 1$, creating a closed curve. Find the formula for the tangent line to the curve at the point $(1, 2, -1)$.
[**Hint:** Think about normal vectors.]

14. Find the radius of the largest circle that can fit inside the closed curve $x^6 + 4y^6 = 1$.

15. Find the maximum and minimum values of the function $f(x, y) = 3x^2 - 2xy + 3y^2 - 2x - 10y$ on the region $x^2 + y^2 \le 9$.