

Math 10C Calculus Practice Midterm I April 16, 2007

Name: _____

ID: _____

Section Hour: _____

TA: _____

Guidelines for the test:

- No books, notes, or calculators are allowed.
- Use the space provided. If necessary, write “see other side” and continue working on the back of the same sheet.
- **Circle your final answers** when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will receive no credit, even if they are correct.
- You have 50 minutes.

Question #	Perfect Score	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. Suppose $f(x)$ is a function whose 5th-degree Taylor polynomial about $x = 0$ is

$$P_5(x) = x - x^2 + x^3 - x^4 + x^5.$$

(a) (5 points) Is f increasing or decreasing at $x = 0$? Explain.

(b) (5 points) What is the concavity of f at $x = 0$? Explain.

(c) (5 points) What is the value of $f^{(4)}(0)$?

(d) (5 points) Suppose you are also given that $f^{(6)}(0) = -72$. Find the 6th-degree Taylor polynomial for $f(x)$ about $x = 0$.

2. Suppose that a is some fixed value with $|a| < 1$. We define the sums S_n as follows:

$$\begin{aligned}S_1 &= a \\S_2 &= a - a^2 \\S_3 &= a - a^2 + a^3 \\S_4 &= a - a^2 + a^3 - a^4 \\&\vdots\end{aligned}$$

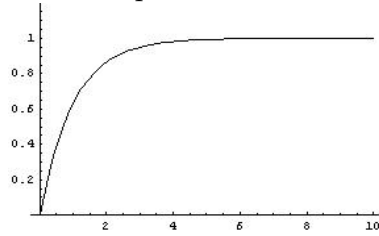
(a) (8 points) Is S_n a geometric series? Explain your answer by referring to features of a geometric series.

(b) (6 points) Suppose now that $a = 2$. What is the value of S_{10} ?

(c) (6 points) Suppose instead that $a = \frac{1}{5}$. What is the value of the infinite series

$$S = a - a^2 + a^3 - a^4 + a^5 - a^6 + a^7 - \dots?$$

3. The graph of $f(x) = 1 - e^{-x}$ is given below. You are informed that $f(x)$ is either a probability density function or a cumulative distribution function for some statistical experiment.



(a) (7 points) If you think $f(x)$ is a density function, sketch a graph for the corresponding distribution function. If you think $f(x)$ is a distribution function, sketch the corresponding density function.

(b) (7 points) Find the mean value for the outcome of the experiment.

(c) (6 points) To what extent is the following statement true? Explain.

“If $p(x)$ is the density function for an experiment and $p(2) = 0.86$, then we can conclude that the outcome $x = 2$ occurs 86% of the time.”

4. Suppose $f(x, y) = 5 - x^2 - y^2$.

(a) (6 points) Describe, in words and with a graph, the cross sections of the surface $z = f(x, y)$ with x fixed.

(b) (6 points) Describe, in words and with a graph, the cross sections of the surface $z = f(x, y)$ with z fixed.

(c) (8 points) Sketch a contour diagram for the function $f(x, y)$ with four labeled contours.

5. (a) (5 points) Find the equation of the linear function $z = c + mx + ny$ whose graph contains the points $(0, 0, 0)$, $(0, 2, -1)$ and $(-3, 0, -4)$.

(b) (5 points) Find the equation of the linear function $z = c + mx + ny$ whose graph intersects the xz -plane in the line $z = 3x + 4$ and intersects the yz -plane in the line $z = y + 4$.

(c) (5 points) Is it possible to determine the equation for a plane, if we know that the points $(7, 1, 2)$, $(8, 3, 6)$ and $(9, 5, 10)$ lie on the plane? Explain.

(d) (5 points) Is it possible to determine the equation for a plane, if we know that the points $(5, 1, 7)$ and $(8, 1, 8)$ lie on the plane, and also that the slope in the x -direction is $\frac{1}{3}$? Explain.

Median and Mean

Let $p(x)$ be a probability density function.

- The median value of x is the value T satisfying

$$\int_{-\infty}^T p(x)dx = 0.5.$$

- The mean value μ of x is given by

$$\mu = \int_{-\infty}^{\infty} xp(x)dx.$$

Geometric Series

$$a + ax + ax^2 + \dots + ax^{n-1} = a \left(\frac{1 - x^n}{1 - x} \right), \text{ if } x \neq 1$$

$$a + ax + ax^2 + ax^3 + \dots = \frac{a}{1 - x}, \text{ if } |x| < 1$$

(If $|x| \geq 1$, then the infinite series diverges.)

Taylor Polynomial

The n^{th} -degree Taylor polynomial for $f(x)$ based at $x = a$ is given by

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

where $f^{(n)}(a)$ is the n^{th} derivative of f , evaluated at a , and $n! = n(n-1)(n-2) \cdots (3)(2)(1)$.