Homework 3 solution and rubric

2.2 6

(a) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ Along $x=0$

$= \lim_{(0,y)\to(0,0)} \frac{0y^3}{0^2+y^6}$

$= \frac{0}{y^6}$

$= 0$

+1.5

(b) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{(y^3,y)\to(0,0)} \frac{y^3 \cdot y^3}{(y^3)^2+y^6} = \frac{1}{2}$ +1.5

(c) From part (a), the limit value of $f(x,y)$ along $x=0$ is 0
From part b, the limit value of $f(x,y)$ along $x=y^3$ is $\frac{1}{2}$

By the definition of a limit of a function of two variables, the limit values along paths $x=0$, $x=y^3$ are the same

$\because \frac{1}{2} \neq 0$

$\therefore f(x,y)$ isn't continous.

+2

12.
(a) $z = f(x_0,y_0) + \left[\frac{\partial f}{\partial x}(x_0,y_0)\right](x-x_0) + \left[\frac{\partial f}{\partial y}(x_0,y_0)\right](y-y_0)$

when $x_0=y_0=0$ $\quad$ ~~$= e^0 + 2e^{2x}$~~ $\quad \frac{\partial f}{\partial x}(x_0,y_0)\ \frac{\partial}{\partial x} e^{2x+3y} = 2e^{2x+3y}$

$\frac{\partial f}{\partial y}(x_0,y_0) = \frac{\partial}{\partial y} e^{2x+3y} = 3e^{2x+3y}$

$z = e^0 + 2e^{2x+3y}(x-0) + 3e^{2x+3y}(y-0)$

when $x=y=0$

$z = 1+2x+3y$

All correct +2 $\quad$ any minor error caused wrong result +1 $\quad$ all wrong +0

(b)

$f(0.1,0) = 1+2(0.1)+3(0) = 1.2$

$f(0,0.1) = 1+2(0)+3(0.1) = 1.3$

All correct +2 $\quad$ either wrong +1 $\quad$ all wrong +0

(c) $f(0.1,0) = e^{2(0.1)+3(0)} = 1.22$

nearly is fine

$f(0,0.1) = e^{2(0)+3(0.1)} = 1.35$

All correct +1 $\quad$ either wrong +0.5

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$z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x-x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y-y_0)$

Each part of z worth 1 point

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(xe^{y^2} - ye^{x^2}\right) = e^{y^2} - 2xye^{x^2}$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(xe^{y^2} - ye^{x^2}\right) = 2xye^{y^2} - e^{x^2}$

when $(x_0, y_0) = (1, 2)$

$\frac{\partial f}{\partial x} = e^4 - 4e \quad \frac{\partial f}{\partial y} = 4e^4 - e$

$\therefore \cancel{z} \quad f(x_0, y_0) = f(1,2) = e^4 - 2e$

$z = \cancel{x(e^4 - 4e)}$

$(e^4 - 2e) + (e^4 - 4e)(x-1) + (4e^4 - e)(y-2)$

$= x(e^4 - 4e) + y(4e^4 - e) + 4e - 8e^4$ +3

(b) $z_2(x_0, y_0) = x_0^2 - y_0^2$

$z_2 = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x-x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y-y_0)$

$= x_0^2 - y_0^2 + 2x_0(x-x_0) - 2y_0(y-y_0)$

$= 2xx_0 - 2yy_0 - x_0^2 + y_0^2$

From part (a) $z = x(e^4 - 4e) + y(4e^4 - e) + 4e - 8e^4$

their coefficient must be same

$\therefore 2x_0 = e^4 - 4e \Rightarrow x_0 = \frac{e^4 - 4e}{2} \quad -2y_0 = 4e^4 - e \Rightarrow y_0 = -\frac{4e^4 - e}{2}$

$z = x_0^2 - y_0^2 = \frac{15e^2 - 15e^8}{4}$

$\therefore$ point is $\left(\frac{e^4 - 4e}{2}, \frac{e - 4e^4}{2}, \frac{15e^2 - 15e^8}{4}\right)$

Correct +2

partical correct +1

23. $z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x-x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y-y_0)$

When $x_0 = 1 \quad y_0 = 2$

$f(x_0, y_0) = 1^2 \cdot 2^3 = 8$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2y^3) = 2xy^3 = 16 \quad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2y^3) = 3x^2y^2 = 12$

$\therefore z = 8 + 16(x-1) + 12(y-2) = -32 + 16x + 12y$ --- +2 get the plane

$\because$ plane z contains $(1,3,20)$ and $(2,1,z)$

$\therefore z = -32 + 16 \cdot 2 + 12 = 12$ --- +1 get the coordinate z

$\therefore l(t) = (1,3,20) + [(1,3,20) - (2,1,12)]t$

$= (1,3,20) + (-1,2,8)t.$ --- +2

answer correct