Homework 4 Solutions

5 (a) Let's state something more general: given a circle C of radius $R > 0$, centered at the origin, a parametrization inducing a counterclockwise orientation, starting at $(R, 0)$, is:

+2

$$c(t) = (R\cos(t), R\sin(t)), \ t \in [0, 2\pi]$$

Thus if $R = 2$, we have

$$c(t) = (2\cos(t), 2\sin(t)), \ t \in [0, 2\pi]$$

this symbol means "belongs to"

(b) If we swap the roles of the x-axis and the y-axis, we see that this is the same question as part a). Thus, we take the answer from a) and swap coordinates:

+2

$$c(t) = (2\sin(t), 2\cos(t)) \quad t \in [0, 2\pi].$$

(c) We can take the answer from either pt a) or b), and just violently shove it up 7 units and to the right 4 units. Thus, for $t \in [0, 2\pi]$

+1

$$c(t) = (2\cos(t) + 4, 2\sin(t) + 7)$$

or

$$c(t) = (2\sin(t) + 4, 2\cos(t) + 7)$$

(6) (a) The direction vector is

$$(1,2,3)-(-2,0,7) = (3,2,-4)$$

Thus we can take either point as our starting point and get

$$\ell(t) = (1,2,3) + t(3,2,-4)$$

+1

or

$$\ell(t) = (-2,0,7) + t(3,2,-4)$$

(You could have subtracted the other way and get $(-3,-2,4)$).

(b) Very straightforward — just try

$$c(t) = (t, t^2) \text{ for } t \in \mathbb{R}$$

+1

c) Consider

$$c(t) = \begin{cases} (0, 4t) & 0 \le t \le 1/4 \\ (4(t-1/4), 1) & 1/4 \le t \le 1/2 \\ (1, 4(t-1/2)) & 1/2 \le t \le 3/4 \\ (4(t-3/4), 0) & 3/4 \le t \le 1 \end{cases} \quad t \in [0,1]$$

Of course this might be complicated, so it's fine if you instead parametrize each piece:

$$C_1(t) = (0, t)$$
$$C_2(t) = (t, 1) \quad \text{for } t \in [0,1]$$
$$C_3(t) = (1, t)$$
$$C_4(t) = (t, 0)$$

+1.5

d) Consider $c(t) = (3\cos(t), 5\sin(t))$ $t \in [0, 2\pi]$.
You can verify that

$$\frac{(3\cos(t))^2}{9} + \frac{(5\sin(t))^2}{25} = 1$$

+1.5

(11) (a) $p = f \circ c$

$$= \begin{bmatrix} 3\sin(t) + 2 \\ \sin(t)^2 + \cos(t)^2 \\ \cos(t) + t^2 \end{bmatrix} = \begin{bmatrix} 3\sin(t) + 2 \\ 1 \\ \cos(t) + t^2 \end{bmatrix}$$ +0.5

By definition of composing functions, just replace the x, y, z w/ the coordinates of c(t)

$$p'(t) = \begin{bmatrix} 3\cos(t) \\ 0 \\ -\sin(t) + 2t \end{bmatrix} \Rightarrow p'(\pi) = \begin{bmatrix} -3 \\ 0 \\ 2\pi \end{bmatrix}$$ +0.5

(b) $c(\pi) = (\cos\pi, \sin\pi, \pi) = (-1, 0, \pi)$ +0.5

$$c'(\pi) = \frac{d}{dt} c(t) \Big|_{t=\pi} = (-\sin(t), \cos(t), 1) \Big|_{t=\pi}$$

$$= (-\sin(\pi), \cos(\pi), 1)$$

$$= (0, -1, 1)$$ +0.5

b)

$$Df(-1,0,\pi) = \begin{bmatrix} \frac{\partial}{\partial x}(3y+2) & \frac{\partial}{\partial y}(3y+2) & \frac{\partial}{\partial z}(3y+2) \\ \frac{\partial}{\partial x}(x^2+y^2) & \frac{\partial}{\partial y}(x^2+y^2) & \frac{\partial}{\partial z}(x^2+y^2) \\ \frac{\partial}{\partial x}(x+z^2) & \frac{\partial}{\partial y}(x+z^2) & \frac{\partial}{\partial z}(x+z^2) \end{bmatrix}_{\substack{(x,y,z)\\=(-1,0,\pi)}}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ 2x & 2y & 0 \\ 1 & 0 & 2z \end{bmatrix}_{\substack{(x,y,z)\\=(-1,0,\pi)}}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 2\pi \end{bmatrix}$$ +2

c) Just multiply matrices.

$$\begin{bmatrix} 0 & 3 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 2\pi \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 2\pi \end{bmatrix}$$ +1

(32) $D(g \circ f)(x, y, z)$

$$= Dg(f(x,y,z)) \cdot Df(x,y,z)$$

$$= \begin{bmatrix} e^u & 0 \\ 1 & \cos v \end{bmatrix}\Bigg|_{\substack{u=xy \\ v=yz}} \cdot \begin{bmatrix} y & x & 0 \\ 0 & z & y \end{bmatrix}$$

+2 +2

$$= \begin{bmatrix} e^{xy} & 0 \\ 1 & \cos(yz) \end{bmatrix} \begin{bmatrix} y & x & 0 \\ 0 & z & y \end{bmatrix}$$

$$= \begin{bmatrix} ye^{xy} & xe^{xy} & 0 \\ y & x + z\cos(yz) & y\cos(yz) \end{bmatrix}$$

If we have $(x,y,z) = (0,1,0)$, then we plug those numbers in and get

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$ +1