Homework 5 solution and rubric

5

2.6 #16

$f_x = \frac{\partial}{\partial x}(f(x,y,z)) = x(1-x^2-y^2)^{-\frac{1}{2}}$

$f_y = \frac{\partial}{\partial y}(f(x,y,z)) = y(1-x^2-y^2)^{-\frac{1}{2}}$ (+1)

$f_z = \frac{\partial}{\partial z}(f(x,y,z)) = -1$

$\therefore \nabla f(x,y,z) = x(1-x^2-y^2)^{-\frac{1}{2}} i + y(1-x^2-y^2)^{-\frac{1}{2}} j - k$

The tangent plane is

$$(1-x_0^2-y_0^2)^{-\frac{1}{2}} x_0(x-x_0) + (1-x_0^2-y_0^2)^{-\frac{1}{2}} y_0(y-y_0) - (z-z_0) = 0$$

$$\Downarrow$$

$$x_0(x-x_0) + y_0(y-y_0) - (1-x_0^2-y_0^2)^{\frac{1}{2}}(z-z_0) = 0$$

$$\therefore T((x_0,y_0), f(x_0,y_0)) = x_0(x-x_0) + y_0(y-y_0) + f(x_0,y_0)(z-z_0)$$
$$= x_0(x-x_0) + y_0(y-y_0) - (1-x_0^2-y_0^2)^{\frac{1}{2}}(z-z_0)$$
$$= 0$$

$\therefore$ the plane tangent to the ~~graph~~ graph of f is orthogonal to the vector (+2)

$\because f(x,y) = -(1-x^2-y^2)^{\frac{1}{2}}$

$\therefore z = -(1-x^2-y^2)^{\frac{1}{2}}$

$\therefore z^2 = 1-x^2-y^2$

$x^2+y^2+z^2=1$

It mean tangent to a sphere is perpendicular to the vector joining that point to origin (+2)

Make sense is OK.

mention sphere (+1)

#20

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= \left(\frac{\partial}{\partial x}(x^2+4y^2-z^2-4), \frac{\partial}{\partial y}(x^2+4y^2-z^2-4), \frac{\partial}{\partial z}(x^2+4y^2-z^2-4\right.$

+2

$= (2x, 8y, -2z)$

This vector is parallel to the plane $2x+2y+z=$
which direction is $(2t, 2t, t)$

$\therefore 2x = 2t, \quad 8y = 2t, \quad -2z = t$

$x = t \quad y = \cancel{\ } \frac{1}{4}t \quad z = -\frac{1}{2}t$

+1

$f(x,y,z) = x^2+4y^2-z^2-4 = 0$

$\therefore t^2 + 4\left(\frac{1}{4}t\right)^2 - \left(\frac{-t}{2}\right)^2 = 4$

$t^2 = 4$

$t = \pm 2$

when $t = 2 \quad (2, \frac{1}{2}, -1)$ +1

when $t = -2 \quad (-2, -\frac{1}{2}, 1)$ +1

#22

(a) $\nabla T = (T_x, T_y, T_z)$ $\quad T(x,y,z) = e^{-x^2-2y^2-3z^2}$

$= \left(\frac{\partial}{\partial x}(T(x,y,z)), \frac{\partial}{\partial y}(T(x,y,z)), \frac{\partial}{\partial z}(T(x,y,z))\right)$

$= \left(-2xe^{-x^2-2y^2-3z^2}, -4ye^{-x^2-2y^2-3z^2}, -6z e^{-x^2-2y^2-3x^2}\right)$ +1

$-\nabla T(1,1,1) = 2e^{-6}i + 4e^{-6}j + 6e^{-6}k$ +1

(b) $e^{8} \cdot \|\nabla T(1,1,1)\| = e^{8} \cdot e^{-6}(\sqrt{(4+16+36)})$

$= 2e^{2}\sqrt{14}$ +1.5

(c) $e^{8} \times |\nabla T(x,y,z)| \leq \sqrt{14}\, e^{2}$

$e^{8} \times |-2xe^{-6}i - 4ye^{-6}j - 6ze^{-6}k| \leq \sqrt{14}\, e^{2}$

$e^{2}(2x+4y+6z) \leq \sqrt{14}\, e^{2}$

$x + 2y + 3z \leq \frac{\sqrt{14}}{2}$

$\left\{ xi + yj + zk,\ x^2+y^2+z^2=1,\ 0 \leq x+2y+3z \leq \frac{\sqrt{14}}{2} \right\}$

+1.5