Name: ______________________

20C, Ferrara, Dec 10, Version 2 SID: ______________________

Final

This exam has 17 problems and 18 pages. Make sure that your exam has all 20 problems and that your name is on every page.

Put your name and student ID on every page.

You must show your work and justify your answers to receive full credit unless otherwise stated.

If you need more space, use the pack of the pages; clearly indicate when you have done this.

You may not use books or calculators on this exam; one hand written 8.5in x 11in page (front and back) of notes is allowed.

Name: ____________________

SID: ____________________

1. (5 points) Where does the line through $(1, 0, 1)$ and $(4, -2, 2)$ intersect the plane $x + y + z = 8$?

2. (5 points) Find an equation of the tangent line to the curve

$$\mathbf{c}(t) = (e^{-t}\sin t, e^{-t}\cos t, e^{-t})$$

at the point $(1, 0, 1)$.

3. (5 points) Find the arc length of the curve $\mathbf{r}(t) = (\frac{1}{2}t^2, \frac{2\sqrt{2}}{3}t^{3/2}, t)$, $0 \leq t \leq 2$.

4. (3 points) Is the function

$$f(x,y) = \begin{cases} \frac{x^4-y^4}{x^2-y^2} & \text{if } (x,y) \neq (0,0) \\ 2 & \text{if } (x,y) = (0,0) \end{cases}$$

continuous? Why or why not?

5. (3 points) Is the function

$$f(x,y) = \begin{cases} \frac{x^4-y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

continuous? Why or why not?

6. (4 points) Is the function

$$f(x,y) = \begin{cases} \frac{x^4-y^4}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

continuous? Why or why not?

7. (10 points) Find the equation of the plane that is tangent to the surface $z = x^2 + y^2$ and perpendicular to the line $\boldsymbol{\ell}(t) = (1 - 3t, 1 - 2t, 2 + 2t)$.

8. (10 points) Let $G(s,t) = (u(s,t), v(s,t))$ and $F(u,v)$ be functions, and let $W = F \circ G$ be the composition of F and G, so $W(s,t) = F(u(s,t), v(s,t))$. Suppose that we know that

$$u(1,0) = 4, v(1,0) = 2, \frac{\partial u}{\partial s}(1,0) = 3, \frac{\partial v}{\partial s}(1,0) = 2$$

$$\frac{\partial u}{\partial t}(1,0) = -5, \frac{\partial v}{\partial t}(1,0) = 6, \frac{\partial F}{\partial u}(4,2) = -2, \frac{\partial F}{\partial v}(4,2) = 3$$

What is $DW(1,0)$?

9. (5 points) Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is paralell to the plane $z = x + y$? If not, explain why, and if so, what are the points?

10. (10 points) Let D be the closed triangular region with vertices $(1, 0), (5, 0), (1, 4)$ and let $f(x, y) = 3 + xy - x - y$. Where does $f(x, y)$ achieve its maximum and minumim values on D are what are the maximum and minimum values?

11. (10 points) Find the volume of the solid S that is bounded by the elliptic paraboloid $2x^2 + y^2 + z = 16$, the planes $x = 2$, $y = 2$, $x = 0$, $y = 0$, and $z = 0$.

12. (10 points) Calculate the integral $\int\int_R xe^{xy} dA$, where $R = [-1, 0] \times [-1, 0]$.

13. (10 points) Let D be the region in the x, y-plane bounded by the lines $y = x, y = 1, x = 0$. Calculate $\int\int_D e^{x/y} dA$.

14. (10 points) Show that if $\mathbf{r}(t) = (x(t), y(t), z(z))$ is a curve such that $\mathbf{r}''(t)$ exists, then

$$\frac{d}{dt}(\mathbf{r}(t) \times \mathbf{r}'(t)) = \mathbf{r}(t) \times \mathbf{r}''(t).$$

(Here the $\times$ denotes the cross product.)

15. (Extra Credit 5 points) Let S be the surface given by $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ where $c > 0$ is a constant. Show that the sum of the $x-$, $y-$, and z-intercepts of any tangent plane to the surface S is the constant c. (Hint: The x-intercept of a surface is the x-value when $y = 0$ and $z = 0$, the y-intercept is the y-value when $x = 0$ and $z = 0$, and the z-intercept is the z-value when $x = 0$ and $y = 0$.)

16. (Extra Credit 5 points) Show that if a curve $\mathbf{r}(t) = (x(t), y(t), z(t))$ is always perpendicular to its tangent vector $\mathbf{r}'(t)$, then the curve $\mathbf{r}(t)$ lies on a sphere with center at the origin.

17. (Extra Credit 5 points) Suppose f is a differentiable function of one variable. Show that all tanget planes to the surface $z = xf(y/x)$ intersect in a common point.