Name: ____________________

20C, Ferrara, Nov 14, Version 2 SID: ____________________

Midterm 2

This exam has BLANK pages and BLANK problems. Make sure that your exam has all BLANK pages and that your name is on every page.
Put your name and student ID on every page.
You must show your work and justify your answers to receive full credit unless otherwise stated.
If you need more space, use the pack of the pages; clearly indicate when you have done this.
You may not use books or calculators on this exam; one hand written 8.5in x 11in page (front and back) of notes is allowed.

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1. (5 points) Consider the following limit

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + 2x^2y^2}{x^3y + y^4}.$$

If the limit exists, compute it. If the limit does not exist, prove that it does not exist. (In either case, you do not need to use ϵ-δ.)

2. (5 points) Find the linear approximation of

$$f(x, y) = x^8y^4 + 6x^3y^2 + 2x^2 + 3x + 2y + 1$$

at the point $(0, 0)$.

3. Let $f(x,y) = \frac{1}{2}x^2 + 5xy + 2y^2 - 3x + 6y$.

   (a) (5 points) Compute the six partial derivatives

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}.$$

   (b) (5 points) Find the critical points of f and determine which are local maximums, which are local minimums, and which are saddle points.

4. (a) (5 points) Calculate the derivative of the function

$$f(x, y) = (4y^2 \sin(x), 2e^x \cos(y))$$

at the point $(0, \frac{\pi}{4})$.

(b) (5 points) Let $g(x, y, z) = (y, \pi z + \pi x)$. Calculate the derivative of $f \circ g$ at the point $(\frac{1}{8}, 0, \frac{1}{8})$.

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5. (5 points) Parametrize the solutions to the equationi $y = x^4$ in $\mathbb{R}^2$ by a curve $c(t)$ that has speed 2 at the point $(0, 0)$, and such that the velocity vector of the curve $c(t)$ at the point $(0, 0)$ has negative x-component.

6. (5 points) Calculate the directional derivative of $f(x, y, z) = yz + xz$ at the point $(1, 1, 1)$ in the direction with positive x-component that is normal to the surface given by $-2x^2 - z^2 + 3y^2 = 4$ at the point $(1, 1, 1)$.