1.) Find integers $s$ and $t$ such that $1 = 5s + 12t$. Show that $s$ and $t$ are not unique.

**Solution:** One choice that works is $s = 5$ and $t = -2$, which we can obtain by using the Euclidean algorithm. Another choice which works is $s = 17 = 5 + 12$ and $t = -7 = -2 - 5$, since $12(5) - 5(12) = 0$, so $s$ and $t$ are not unique.

2.) Determine $4^{2013}$ mod 9. Justify your answer.

**Solution:** Since $4^3 = 64 = 1 \mod 9$, we have that $4^{2013} = (4^3)^{671} = (1)^{671} \mod 9 = 1$.

3.) In $Z_{18}$, find all: (1) generators, (2) elements of order 6. Justify your answer.

**Solution:** (1) Recall that $Z_{18} = \{0, 1, 2, \ldots, 17\}$ under addition modulo 18. The generators of $Z_{18}$ will be those elements which are relatively prime to 18, so the set of generators is: \{1, 5, 7, 11, 13, 17\}.

(2) The order of an element $a \in Z_n$ is $|a| = n/gcd(a, n)$. If $a \in Z_{18}$ has order 6, then $|a| = 18/gcd(a, 18) = 6$, so $gcd(a, 18) = 3$. Thus the elements of order 6 are: \{3, 15\}.

4.) List all subgroups of $U(7)$. Justify your answer.

**Solution:** Recall that $U(7)$ is the set \{1, 2, 3, 4, 5, 6\} under multiplication modulo 7. We compute the cyclic subgroups of $U(7)$ by computing the powers of each element:
Since $U(7)$ is cyclic (we see from above that both 3 and 5 are generators), we know that every subgroup is a cyclic subgroup. Thus we have found all subgroups of $U(7)$.

5.) Let $G$ be a cyclic group of order 21 and $a$ is a generator of $G$. What is the order of the subgroup $< a^{14} > \cap < a^{15} >$? Justify your answer.

Solution: The answer is 1. In the cyclic group generated by $a$, where $|a| = n$, we know that $< a^k > = < a^{\gcd(k,n)} >$. Thus $< a^{14} > = < a^7 > = \{a^7, a^{14}, a^0 = e\}$. We also compute $< a^{15} > = < a^3 > = \{a^3, a^6, a^9, a^{12}, a^{15}, a^{18}, a^{21} = e\}$. We see then that $< a^{14} > \cap < a^{15} > = \{e\}$, which has order 1.