(1) Prove that every vector space $V$ contains a unique zero vector $0$. Thus it is correct to refer to $0$ as “the” zero vector of a given vector space.

(2) Let $V$ be a vector space, and let $S$ be a subset of $V$.
   (a) Prove that if $S$ contains the zero vector, then it is linearly dependent.
   (b) Prove that if $S$ contains a linearly dependent set, then it is linearly dependent.
   (c) Prove that if $S$ is contained in a linearly independent set, then it is linearly independent.

(3) (a) Let $V$ be the set of all polynomials of degree $n$ in a variable $x$. Explain why $V$ is not a vector space.
    (b) Let $V$ be the vector space of all polynomials of degree less than $n$ in a variable $x$. Prove that $V$ is $n$-dimensional.

(4) (a) Let $V$ be the vector space of all functions $f : \mathbb{R} \to \mathbb{R}$. Prove that $V$ is infinite-dimensional.
    (b) Let $V$ be a vector space. What is the dimension of $\text{Fun}(V,V)$, the vector space of functions $f : V \to V$?

(5) Let $V$ be an $n$-dimensional vector space. Prove that $V$ contains a subspace of dimension $k$ for each $0 \leq k \leq n$. 