“The calculations necessary are not performed in the mind but in the blood. They are like those vestibular reckonings performed in the inner ear for standing upright.”

—from *The Stonemason*, by Cormac McCarthy.

(1) (a) Let $m_n$ denote the number of simple graphs with vertex set $V = \{v_1, \ldots, v_n\}$. Find a formula for $m_n$, and calculate the numbers $m_1, m_2, m_3, m_4$.

(b) A graph is said to be connected if there exists a walk between any pair of vertices. Let $c_n$ denote the number of connected simple graphs with vertex set $\{v_1, \ldots, v_n\}$. Find a recursive formula for $c_n$, and calculate $c_1, c_2, c_3, c_4$.

(2) Let $K_n$ be the complete graph on $V = \{v_1, \ldots, v_n\}$. Using combinatorial reasoning (i.e. no matrix algebra), find a formula for the number of $r$-step walks on $K_n$ from $v_1$ to $v_1$.

(3) Let $K_{m,n}$ be the complete bipartite graph on $V = \{v_1, \ldots, v_m, w_1, \ldots, w_n\}$, i.e. the simple graph with this vertex set in which $v_i$ and $w_j$ are adjacent for all $1 \leq i \leq m$ and $1 \leq j \leq n$, and there are no other edges. Find a formula for the total number of closed $r$-step walks on $K_{m,n}$.

(4) Culture: browse through sections II.1, II.2, II.3 of “Methods of Modern Mathematical Physics” by Michael Reed and Barry Simon.