“I know things I ain’t never studied. I know things I ain’t never even thought about.”

— from *Outer Dark*, by Cormac McCarthy.

(1) Let $\Gamma$ be a graph on the vertex $V = \{v_1, \ldots, v_n\}$, where $n \geq 2$, and let $\Gamma'$ be the induced subgraph on $V' = \{v_1, \ldots, v_{n-1}\}$. Prove that the eigenvalues of $\Gamma$ and $\Gamma'$ interlace.

(2) Determine the eigenvalues of the complete bipartite graph $K_{m,n}$.

(3) Stanley, 2.1.

(4) Stanley, 2.5.