

## QUAL PREP SESSION 2

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### Problem 1

Suppose  $E \subset \mathbb{R}$  is a measurable set such that  $m(E) = 1$ , find a measurable set  $A \subset E$  such that  $m(A) = \frac{1}{2}$ .

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### Problem 2

Suppose  $f: \mathbb{R} \rightarrow \mathbb{R}$  is  $L^1$ . Prove

$$\lim_{t \rightarrow 0} \int |f(x+t) - f(x)| dx = 0$$

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### Problem 3

Prove the Riemann-Lebesgue lemma:

Given any  $f \in L^1([-\pi, \pi])$ , then

$$\lim_{\lambda \rightarrow \infty} \int_{-\pi}^{\pi} f(x) e^{i\lambda x} dx = 0$$

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