HWK #1, DUE WEDNESDAY 10/16

1. Hartshorne I, 1.1–7.
2. Show that if $k \geq 2$ is any natural number and $V \subset \mathbb{A}^n_K$ is a subset with at most kn points in linear general position then we may find polynomials of degree k whose zero locus is V.

Challenge Problems:

3. Show that every square matrix over a field satisfies its characteristic polynomial. (*Hint: reduce to the case of an algebraically closed field. Show that the space of all matrices is naturally a variety and identify a large subset where the result is obvious*).
4. An **analytic subvariety** is a subset V of $\mathbb{C}^n$ such that locally, in the Euclidean topology (that is, not the Zariski topology), V is defined by holomorphic functions. Give an example of an analytic variety of $\mathbb{C}^n$ which is not an algebraic subvariety of $\mathbb{A}^n_{\mathbb{C}}$.