HOMEWORK #7, DUE WEDNESDAY DECEMBER 3RD

1. How many roots does the equation
$$z^7 - 2z^5 + 6z^3 - z + 1 = 0,$$
have in the disk $|z| < 1$? (*Hint:* look for the biggest term when $|z| = 1$ and apply Rouché's Theorem.)

2. How many roots of the equation
$$z^4 - 6z + 3 = 0,$$
have their modulus between 1 and 2?

3. Evaluate the following integrals, using the methods of residues.

(i)
$$\int_0^{\pi/2} \frac{dx}{a + \sin^2 x} \qquad |a| > 1.$$

(ii)
$$\int_0^\infty \frac{x^2 \, \mathrm{d}x}{x^4 + 5x^2 + 6}.$$

(iii)
$$\int_0^\infty \frac{x^2 \, \mathrm{d}x}{(x^2 + a^2)^3}, \qquad a \text{ real.}$$

(iv)
$$\int_0^\infty (1 + x^2)^{-1} \log x \, \mathrm{d}x.$$