HOMEWORK #8, DUE WEDNESDAY DECEMBER 10TH

1. Show that the Laurent series for the function

$$\frac{1}{e^z - 1}$$

at the origin has the form

$$\frac{1}{z} - \frac{1}{2} + \sum_{k=1}^{\infty} (-1)^{k-1} \frac{B_k}{(2k)!} z^{2k-1}.$$

Calculate B_1, B_2 and B_3.
2. Find expressions for the Taylor series of $\tan z$ and the Laurent series of $\cot z$ in terms of the Bernoulli numbers $B_1, B_2, \ldots$.
3. Comparing coefficients in the Laurent developments of $\cot \pi z$ and of its expression as a sum of partial fractions, find the values of

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^4} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{1}{n^6}.$$

4. Express

$$\sum_{n=-\infty}^{\infty} \frac{1}{z^3 - n^3}$$

in closed form.
Postponed:
5. Show that

$$\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}.$$

6. What is the genus of $\cos \sqrt{z}$?