FIRST HOMEWORK, DUE WEDNESDAY APRIL 8TH

1. Show that the divisor of zeroes minus poles associated to the rational n-form

$$\omega = \frac{dx_1}{x_1} \wedge \frac{dx_2}{x_2} \wedge \cdots \wedge \frac{dx_n}{x_n},$$

on $\mathbb{P}^n$ is

$$-(H_0 + H_1 + H_2 + \cdots + H_n),$$

by filling in the gap in the notes.
2. Let C be a smooth curve of genus g. Show that $h^0(C, \mathcal{O}_C(K_C)) = g$ and $\deg K_C = 2g - 2$.
3. Determine the genus g of a smooth curve C of degree d in $\mathbb{P}^2$.
4. Let C be a smooth curve of degree d in $\mathbb{P}^2$ and suppose that we project C down to a line from a general point in $\mathbb{P}^2$. How many ramification points are there? (You may assume that all ramification is simple, that is, there is exactly one ramification point lying over every branch point, where the local ramification is two).
5. Classify all pairs (C, Δ) where

- C is a smooth curve,
- the coefficients of Δ are of the standard form $(r-1)/r$ where r is a positive integer (including ∞, that is, including coefficient one) and,
- the degree of $K_C + \Delta$ is at most zero.

6. What is the smallest degree of a pair (C, Δ) such that

- C is a smooth curve,
- the coefficients of Δ are of the standard form $(r-1)/r$ where r is a positive integer (including ∞, that is, including coefficient one) and
- the degree of $K_C + \Delta$ is positive?

Which pair achieves this bound?
7. Show that the cardinality of the automorphism group of a smooth curve B of genus g is at most $84(g-1)$. (*Hint* Let $C = B/G$ where G is the automorphism group. Relate the degree of K_B with the degree of a divisor on C, using the form of Riemann-Hurwitz given in class.
Challenge problems:
8. Can you write down curves which achieve the upper bound of $84(g-1)$ (note that for some genera there are in fact no such curves).

9. Give an improved general upper bound on the order of the automorphism group, for those curves C of genus g, where the order is less than $84(g-1)$.
10. What is your best **guess** as to how small the self-intersection of $K_S + \Delta$ can be, given that

- S is a smooth surface,
- the coefficients of Δ are of the standard form $(r-1)/r$ where r is a positive integer (including ∞, ie including coefficient one).
- $K_S + \Delta$ is ample.

11. Same question, in dimension n.