## FIFTH HOMEWORK, DUE WEDNESDAY MAY 21ST

1. Let $S$ be a smooth surface and let $\nu$ be any valuation. Show that the log discrepancy of $\nu$ with respect to $K_S$ is at least two, with equality if and only if $\nu$ is the blow up of a point.
2. Let $(X, \Delta)$ be a log pair of log discrepancy at most zero. Define the anti-log discrepancy $b$ to be the supremum of the log discrepancies of valuations of log discrepancy at most zero,

$$b = \sup_{a(\nu,X,\Delta)\leq 0} a(\nu, X, \Delta).$$

Show that the anti-log discrepancy is in fact a maximum, that is, show that there is a valuation $\nu$ such that $a(\nu, X, \Delta) = b$.
3. Let $(X, \Delta)$ be a log pair and let $D \geq 0$ be any divisor. The log canonical threshold of $D$ with respect to $(X, \Delta)$ is the largest real number such that $(X, \Delta + D)$ has log discrepancy at least zero.
Let $C = C_n$ be the plane curve $(y^2 + x^n = 0) \subset \mathbb{C}^2$ and let $\lambda = \lambda_n$ be the log canonical threshold of $C$ with respect to $(\mathbb{A}^2, 0)$, so that $(\mathbb{A}^2, \lambda C)$ has log discrepancy zero.
(i) What is $\lambda_2$?
(ii) What is $\lambda_3$?
(iii) What is $\lambda_4$?
(iv) Can you guess the log canonical threshold when $C = C_{n,m}$ is the plane curve given by $y^m + x^n$?
4. Let $S$ be a surface over $\mathbb{C}$ which is invariant under complex conjugation (in other words, suppose that $S$ is a surface over $\mathbb{R}$). Classify the $K_S$-extremal rays which are invariant under complex conjugation (in other words, classify the $K_S$-extremal rays of the real structure).