

## PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:
  - (i) a left coset.
  - (ii) the index of a subgroup.
  - (iii) the Cartesian product of two sets.
  - (iv) the direct product of two groups.
  - (v) an isometry of the plane.
  - (vi) a group homomorphism.
  - (vii) image of a subgroup.
  - (viii) inverse image of a subgroup.
  - (ix) the kernel.
  - (x) a normal subgroup.
  - (xi) a factor group.
  - (xii) automorphism of a group.
2. If  $H \times G$  is the direct product of two groups  $H$  and  $G$ ,  $h \in H$  has order  $m$  and  $g \in G$  has order  $n$  then what is the order of  $(h, g) \in H \times G$ ?
3. State the fundamental theorem of finitely generated abelian groups. Find all abelian groups of order 216.
4. If  $\phi: \mathbb{Z} \rightarrow S_7$  is a group homomorphism such that
$$\phi(1) = (1, 7)(1, 6)(1, 5)(1, 4)(1, 3)(1, 2)$$
then what is  $\phi(100)$ ?
5. Find a homomorphism  $S_4 \rightarrow \mathbb{Z}_2$  which is onto.
6. Is there a homomorphism  $S_4 \rightarrow \mathbb{Z}_5$  which is onto? Explain.