PRACTICE PROBLEMS FOR THE 2ND MIDTERM

1. Give the definition of:
   (i) a ring.
   (ii) the product of two rings.
   (iii) a ring homomorphism.
   (iv) a commutative ring.
   (v) a ring with unity.
   (vi) a unit.
   (vii) a field.
   (viii) a zero-divisor.
   (ix) the cancellation laws.
   (x) an integral domain.
   (xi) the characteristic of a ring.
   (xii) the Euler phi-function.
   (xiii) field of fractions.

2. Classify the following group according to the fundamental theorem of finitely generated abelian groups
   \[
   \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_4 \langle (3, 0, 0) \rangle.
   \]

3. Give an example of a group \( G \) having no elements of finite order \( > 1 \) but having a quotient group \( G/H \), all of whose elements are of finite order.

4. Determine whether the given operations are defined on the set and whether they give a ring. If we don’t get a ring, explain why. If we do get a ring, state whether the ring is commutative, whether it has unity and whether it is a field.
   (i) \( 2\mathbb{Z} \times \mathbb{Z} \) with the usual addition and multiplication.
   (ii) The set of all pure imaginary complex numbers
       \[
       \{ ri \mid r \in \mathbb{R} \}
       \]
       with the usual addition and multiplication.

5. Describe all units in the ring \( \mathbb{Z} \times \mathbb{Q} \times \mathbb{R} \).

6. Show that the set of all units \( U \) in a ring \( R \) is a group under multiplication.

7. Let \( R \) be a commutative ring with \( 1 \neq 0 \). Show that \( R \) is an integral domain if and only if \( R \) satisfies the cancellation laws.

8. Show that \( 2^{11,213} - 1 \) is not divisible by 11.