

HOMEWORK 7, DUE TUESDAY NOVEMBER 22ND

1. For Chapter 2, Section 9: 1, 2, 3.
2. Let H and K be two normal subgroups of a group G , whose intersection is the trivial subgroup. Prove that every element of H commutes with every element of K . (*Hint. Consider the commutator of an element of H and an element of K .*)
3. Prove that a group G is isomorphic to the product of two groups H' and K' if and only if G contains two normal subgroups H and K , such that
 - (1) H is isomorphic to H' and K is isomorphic to K' .
 - (2) $H \cap K = \{e\}$.
 - (3) $G = H \vee K$.
4. **Challenge Problem.** Find an example of a finite set, together with a binary operation, which satisfies all the axioms for a group, except associativity.