# PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:

(i) a group.
(ii) an abelian group.
(iii) the order of a group.
(iv) a subgroup.
(v) a proper subgroup.
(vi) closed under multiplicaton.
(vii) closed under inverses.
(viii) an equivalence relation.
(ix) an equivalence class.
(x) a partition.
(xi) a left coset.
(xii) the index of a subgroup.
(xiii) the subgoup generated by a subset $S$.
(xiv) a finitely generated group.
(xv) a cyclic group.

2. Show that a subset $H$ of a group $G$ is a subgroup if and only if $H$ is non-empty, closed under multiplication and closed under inverses.

3. Let $G$ be a group and let $H$ and $K$ be two subgroups.

(i) Show that the intersection $H \cap K$ is a subgroup.

(ii) Is the union $H \cup K$ a subgroup?

4. Give a description of the symmetry group of the square $D_4$ and find all of its subgroups. Pick one subgroup of each order and find its left cosets.

5. Prove Lagrange's Theorem.

6. Show that every group of order a prime is abelian.