1. Give the definition of:
   (i) a group.
   (ii) an abelian group.
   (iii) the order of a group.
   (iv) a subgroup.
   (v) a proper subgroup.
   (vi) closed under multiplication.
   (vii) closed under inverses.
   (viii) an equivalence relation.
   (ix) an equivalence class.
   (x) a partition.
   (xi) a left coset.
   (xii) the index of a subgroup.
   (xiii) the subgroup generated by a subset $S$.
   (xiv) a finitely generated group.
   (xv) a cyclic group.

2. Show that a subset $H$ of a group $G$ is a subgroup if and only if $H$
   is non-empty, closed under multiplication and closed under inverses.

3. Let $G$ be a group and let $H$ and $K$ be two subgroups.
   (i) Show that the intersection $H \cap K$ is a subgroup.
   (ii) Is the union $H \cup K$ a subgroup?

4. Give a description of the symmetry group of the square $D_4$ and find
   all of its subgroups. Pick one subgroup of each order and find its left
   cosets.

5. Prove Lagrange’s Theorem.

6. Show that every group of order a prime is abelian.