

PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:

- (i) a proposition.
- (ii) a predicate.
- (iii) contrapositive.
- (iv) the absolute value.
- (v) divides.
- (vi) even.
- (vii) odd.
- (viii) a graph.
- (ix) an edge.
- (x) the degree of a vertex.
- (xi) a walk.
- (xii) a trail.
- (xiii) an Euler trail.

2. Prove that

$$|c| + |d| \geq |c + d|,$$

for all real numbers c and d .

3. Prove that an integer n is odd if and only if there is an integer k such that $n = 2k + 1$.

4. Prove that $n(n + 1)$ is even for every integer.

(a) Using (3).

(b) Using induction (prove the result for every positive integer first and then use the identity $-m(-m + 1) = l(l + 1)$, where $l = m - 1$, to conclude for all integers).

5. Let x and y be positive real numbers. Show that the geometric mean is always at most the arithmetic mean

$$\sqrt{xy} \leq \frac{x + y}{2},$$

with equality if and only if $x = y$.

6. Prove that for all integers $n \geq 2$,

$$1 + 4 + 18 + 96 + 600 + \cdots + (n - 1)!(n - 1) + n!n = (n + 1)! - 1.$$