PRACTICE PROBLEMS FOR THE SECOND MIDTERM

1. Give the definition of:
   (i) the factorial of an integer.
   (ii) the Fibonacci sequence.
   (iii) the Golden ratio.
   (iv) equality of two sets.
   (v) a subset.
   (vi) the emptyset.
   (vii) union; intersection; difference; the symmetric difference.
   (viii) the power set.
   (ix) monotonic increasing sequence; monotonic decreasing sequence; monotonic sequence.
   (x) upper bound of a set; lower bound of a set.
   (xi) infimum of a set; supremum of a set.
   (xii) function.
   (xiii) composition of functions.
   (xiv) identity function.
   (xv) injective; surjective; bijective;
   (xvi) inverse function.
2. Find the powerset of
   \{1, 2, \{1\}, \{1, 2\}\}.
3. Let \(A\), \(B\) and \(C\) be three sets. If
   \(A \cap B \subseteq A \cap C\) and \(A \cup B \subseteq A \cup C\)
   then prove that \(B \subseteq C\).
4. Let \(A\), \(B\) and \(C\) be three sets. Prove that
   \(A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\).
5. Prove or disprove:
   \(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \forall z \in \mathbb{R}, x + y = z\).
6. Prove or disprove:
   (a) \(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, -x^4 < y\).
   (b) \(\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, -x^4 < y\).
(c) \[ \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, -x^3 < y. \]

(d) \[ \exists y \in \mathbb{R}, \forall x \in \mathbb{R}, -x^3 < y. \]

7. Let \( A \subset \mathbb{Z} \). Translate

“A has a maximum”

into a statement that uses only symbols and quantifiers. Negate the statement. Find an example of a set \( A \) where the statement is true and another set where the statement is false.

8. Let \( f: A \to B \) be a function. Prove that

(a) \( f \) is injective if and only if either \( A \) is the emptyset or there is a function \( g: B \to A \) such that \( g \circ f = \text{id}_A: A \to A \).

(b) \( f \) is surjective if and only if there is a function \( g: B \to A \) such that \( f \circ g = \text{id}_B: B \to B \).