HOMEWORK 5, DUE WEDNESDAY FEBRUARY 15TH

1. Let R be an integral domain. Let a and b be two elements of R. Show that if d and d' are both a gcd for the pair a and b, then d and d' are associates.
2. Let R be a UFD.
(a) Prove that for every pair of elements a and b of R, we may find an element $m = [a, b]$ that is a **least common multiple**, that is

(1) $a|m$ and $b|m$,
(2) and if $a|m'$ and $b|m'$ then $m|m'$.

Show that any two lcm's are associates.
(b) Show that if (a, b) denotes the gcd then $(a, b)[a, b]$ is an associate of ab.
3. Chapter 4, §5: 3(a), (d).
4. Find the greatest common divisor of $135 - 14i$ and $155 + 34i$ in the ring of Gaussian integers $\mathbb{Z}[i]$.