MODEL ANSWERS TO THE SEVENTH HOMEWORK

1. (i) Probably the easiest example is to take the zero ideal in $\mathbb{Z}$. This is prime, as $\mathbb{Z}$ is an integral domain, but it is not maximal as the quotient, $\mathbb{Z}$, is not a field.
(ii) Take the example given in (iii).
(iii) By Gauss' Lemma, $\mathbb{Z}[x]$ is a UFD. On the other hand I claim the ideal $I = \langle 2, x \rangle$ is not principal. Indeed suppose it was, so that $\langle 2, x \rangle = \langle f(x) \rangle$. As $2 \in I$ it follows that $f(x)$ divides 2. Up to associates, it would then follow that $f(x) = 1$ or 2. By the same token, $f(x)$ must divide x as well, and so $f(x) = 1$. But this is a contradiction, as $1 \notin I$.