PRACTICE PROBLEMS FOR THE SECOND MIDTERM

1. Give the definition of:

(i) associates of an integral domain.
(ii) an irreducible element of an integral domain.
(iii) a prime element of an integral domain.
(iv) unique factorisation domain.
(v) principal ideal domain.
(vi) Euclidean domain.
(vii) a partial order.
(viii) a total order.
(ix) the ascending chain condition.
(x) the gcd of a pair of elements of an integral domain.
(xi) the content of a polynomial over a UFD.

2. Let R be an integral domain.
(i) Describe the factorisation algorithm.
(ii) Show that the factorisation algorithm always terminates if and only if the set of principal ideals satisfies the ACC.

3. Let R be an integral domain. Show that every element of R has at most one factorisation into primes, up to order and associates.

4. Show that the set of all ideals satisfies the ACC in a PID.

5. (i) Prove that $F[x]$ is a Euclidean domain, where F is a field.
(ii) Show that a polynomial $f(x) \in F[x]$ has a linear factor if and only if it has a zero.
(iii) Show that a polynomial of degree two or three is irreducible if and only if it does not have any zeroes.

6. (i) State Gauss' Lemma.
(ii) Show that if R is a UFD then so is $R[x]$.
(iii) Show that if R is a UFD then so is $R[x_1, x_2, \dots, x_n]$.