14. Reflections of waves

Now we look at the same problem for the wave equation. We start with the Dirichlet boundary problem

\[ v_{tt} = c^2 v_{xx} \quad \text{for} \quad 0 < x < \infty, \quad 0 < t < \infty \]
\[ v(x, 0) = \phi(x) \quad v_t(x, 0) = \psi(x) \quad \text{for} \quad t = 0 \]
\[ v(0, t) = 0 \quad \text{for} \quad x = 0. \]

We employ the same trick as before. Let \( \phi_{\text{odd}} \) and \( \psi_{\text{odd}} \) be the odd extensions of \( \phi \) and \( \psi \) to the whole real line. Let \( u(x, t) \) be the solution to the wave equation on the whole real line and let \( v \) be the restriction of \( u \) to positive values of \( x \).