HOMEWORK 3, DUE WEDNESDAY APRIL 22ND, 12PM

1. Derive the formula

$$\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2}\,\mathrm{d}x = \frac{\pi}{2}(b-a) \qquad \text{where} \qquad a \geq 0, b \geq 0.$$

Use this to calculate

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,\mathrm{d}x,$$

using the identity

$$2\sin^2 x = 1 - \cos(2x).$$

2. Calculate

$$\int_0^\infty \frac{\mathrm{d}x}{\sqrt{x}(x^2+1)}$$

(i) Using an indented path.
(ii) Using a keyhole contour.

3. Calculate

$$\int_0^\infty \frac{1}{(x^2+1)(x+1)}\,\mathrm{d}x$$

4. Derive the formula

$$\int_0^\infty \frac{\sqrt[3]{x}\,\mathrm{d}x}{(x+a)(x+b)} = \frac{2\pi}{\sqrt{3}}\frac{\sqrt[3]{a} - \sqrt[3]{b}}{a-b} \qquad \text{where} \qquad a > b > 0.$$

Hint: Use a keyhole contour.

5. Calculate

$$\int_0^\infty \frac{(\ln x)^2}{x^2+1}\,\mathrm{d}x.$$

You will need the value of the integral

$$\int_0^\infty \frac{1}{x^2+1}\,\mathrm{d}x$$

and you will compute

$$\int_0^\infty \frac{\ln x}{x^2+1}\,\mathrm{d}x$$

on the way.