HOMEWORK 4, DUE WEDNESDAY APRIL 29TH,
12PM

1. Show that $z^4 + 2z^2 - z + 1$ has exactly one root in each quadrant.
2. How many roots does $z^4 + z^3 + 4z^2 + 3z + 2$ have in each quadrant?
3. Find the number of roots of $z^6 + 4z^4 + z^3 + 2z^2 + z + 5$ in the first quadrant.
4. Find the number of roots of $z^4 + 4z^3 + 4z^2 + az + 3$ such that Re($z$) < 0.
5. Show that $2z^5 + 6z - 1$ has one root in the interval (0, 1) and four roots in the annulus
   \[ \{ z \in \mathbb{C} \mid 1 < |z| < 2 \} \].
6. Show that if $m$ and $n$ are positive integers then the polynomial
   \[ p(z) = 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^n}{n!} + 3z^n \]
   has exactly $n$ roots in the unit disk $\Delta$.
7. If $\lambda$ is a complex number such that $|\lambda| < 1$, then
   \[ (z - 1)^n e^z + \lambda(z + 1)^n \]
   has $n$ zeroes in the right hand plane, Re($z$) > 0. Show these zeroes are
   simple if $\lambda \neq 0$.

Challenge Problems: (Just for fun)

8. Let $U$ be a bounded domain and let $f(z)$ and $h(z)$ be two meromorphic functions on $U$ that are holomorphic on $\partial U$. Suppose that
   \[ |h(z)| < |f(z)| \]
on $\partial U$.
   (i) Give an example where $f(z)$ and $f(z) + h(z)$ have a different number of zeroes on $U$.
   (ii) What comparison can we make between $f(z)$ and $f(z) + h(z)$? Prove your assertion.