HOMEWORK 0, DUE TWELTH OF NEVER

1. Identify and sketch the set of points satisfying:
   (a) $|z - 1 - i| = 1$.
   (b) $1 < |2z - 6| < 2$.
   (c) $|z - 1|^2 + |z + 1|^2 < 8$.
   (d) $|z - 1| + |z + 1| < 2$.
   (e) $|z - 1| < |z|$.
   (f) $0 < \text{Im } z < \pi$.
   (g) $-\pi < \text{Re } z < \pi$.
   (h) $|\text{Re } z| < |z|$.
   (i) $\text{Re}(iz + 2) > 0$.
   (j) $|z - i|^2 + |z + i|^2 < 2$.

2. Verify the following identities, from the definitions:
   (a) $\overline{z + w} = \overline{z} + \overline{w}$.
   (b) $\overline{zw} = \overline{z} \overline{w}$.
   (c) $|\overline{z}| = |z|$.
   (d) $|\overline{z}|^2 = z \overline{z}$.

   Draw sketches to illustrate (a) and (c).

3. Fix a real number $\rho > 0$, $\rho \neq 1$ and fix $z_0$ and $z_1 \in \mathbb{C}$. Show that the set
   $|z - z_0| = \rho |z - z_1|$
is a circle. Sketch this circle for \( \rho = 1/2 \) and 2 when \( z_0 = 0 \) and \( z_1 = 1 \). What happens when \( \rho = 1 \)?

**Challenge Problems:** (Just for fun)

4. Let 

\[
f = f_0 : \mathbb{R} \rightarrow \mathbb{R}
\]

be the function given by the rule

\[
f(x) = \begin{cases} 
e^{-1/x^2} & x \neq 0 \\ 0 & x = 0, \end{cases}
\]

Show that there are polynomials \( p_n(x) \) and \( q_n(x) \) such that the function 

\[
f_n : \mathbb{R} \rightarrow \mathbb{R}
\]

given by the rule

\[
f_n(x) = \begin{cases} \frac{p_n(x)}{q_n(x)} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0, \end{cases}
\]

is continuous and its derivative is \( f_{n-1} \).