1. Let \(a\) be a complex number and let \(\rho > 0\) be a positive real. Show that the equation
\[
|z|^2 - 2 \text{Re}(\bar{a}z) + |a|^2 = \rho^2
\]
represents a circle centred at \(a\) with radius \(\rho\).

2. Consider the polynomial \(p(z) = z^3 + z^2 + z + 1\).
   (a) Verify that \(i\) is a root of \(p(z)\).
   (b) Find the other roots.

3. For which integers \(n\) is \(i\) an \(n\)th root of unity?

4. Let \(n \geq 1\) be an integer.
   (a) Show that
   \[
   1 + z + z^2 + z^3 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.
   \]
   (b) Show that
   \[
   1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n + \frac{1}{2})\theta}{2 \sin \frac{\theta}{2}}.
   \]

5. Show that
\[
\left(1 + i \tan \frac{\theta}{2}\right)^n = \frac{1 + i \tan n\theta}{1 - i \tan n\theta}
\]
for every integer \(n\).

6. Find the distinct sixth roots of \(-64\).

7. Show that
\[
(1 - \sqrt{3}i)^{10} = 2^9(-1 + \sqrt{3}i).
\]

**Challenge Problems:** (Just for fun)

8. Prove the triangle inequality
\[
|z + w| \leq |z| + |w|,
\]
with equality if and only if either \(z = 0\) or \(w\) is a positive real scalar multiple of \(z\).