HOMEWORK 4, DUE FRIDAY FEBRUARY 7TH, 12PM

1. Let s be the sum of the alternating harmonic series:

$$s = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \dots.$$

Show that the sum of the series

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \dots$$

converges to $3s/2$ (*Hint: organise the terms of the alternating harmonic series into groups of* 4 *and compare them with groups of* 3 *in the second series.*)

2. Show that

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

diverges while

$$\sum_{n=2}^{\infty} \frac{1}{n \log^2 n}$$

converges.

3. Find power series centred at the given point for the following functions and identify the radius of convergence:

(a)

$$e^{2z}$$

centred around the origin.

(b)

$$2\cos z - 3\sin z$$

centred around the origin.

(c)

$$\sin z^2$$

centred around the origin.

(d)

$$\frac{1}{3 - 2z}$$

centred around the origin.

(e)

$$\frac{1}{1 - z^2}$$

centred around the origin.

(f)

$$\frac{2z-5}{6-5z+z^2}$$

centred around the origin.

(g)

$$\frac{1}{1-z}$$

centred around i.

Challenge Problems: (Just for fun)

4. Check that the two parabolae,

$$xy = a \qquad \text{and} \qquad x^2 - y^2 = b$$

where a and b are real constants intersect at right angles.

5. Show that the alternating hamronic series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots$$

converges, by showing that if s_n is the nth partial sum

$$s_n = \sum k = 1^n \frac{(-1)^{k+1}}{k}$$

then

$$S_2 < S_4 < S_6 < \cdots < S_5 < S_3 < S_1.$$