Math 20D (J. Oliver)
Practice Exam 2

Problem 1

Consider the ODE:

\begin{align*}
y'' - 25y &= 0.
\end{align*}

(a) Find the solution to the initial value problem (1) with initial conditions:

\begin{align*}
y(0) &= 0, & y'(0) &= 1.
\end{align*}

(b) Let \( y_1(t) \) and \( y_2(t) \) be defined by:

\begin{align*}
y_1(t) &= e^{5t} + e^{-5t}, & y_2(t) &= 10(e^{5t} + e^{-5t}).
\end{align*}

Do \( y_1, y_2 \) form a fundamental set of solutions to equation (1)? Why or why not?

Problem 2

(a) Write the second order ODE as a system of first order equations in standard form:

\begin{align*}
t^2y'' + 3y' - e^ty &= 0.
\end{align*}

(b) Transform the following system into a single second order ODE:

\begin{align*}
x_1' &= 2x_2, & x_2' &= -2x_1, & x_1(0) &= 3, & x_2(0) &= 4.
\end{align*}

Problem 3

Consider the ODE:

\begin{align*}
y'' + 4y &= t^2.
\end{align*}

(a) Write the guess for the particular solution \( y_p \) using the method of undetermined coefficients. Do not determine the coefficients. You do not need to show work for this part.

(b) Find the general solution to Eq. (2). You may use any method taught in this class.

Problem 4

Suppose the vectors: \( \mathbf{x}_1(t) = \langle t, 1 \rangle, \mathbf{x}_2(t) = \langle e^{2t}, e^{2t} \rangle \) solve the system:

\begin{align*}
\dot{\mathbf{x}}(t) &= \mathbf{P}\mathbf{x}(t),
\end{align*}

with \( \mathbf{P}(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} \).

Find \( a, b, c, d \).