1. Consider the experiment of tossing two fair coins. Let $A$ be the event that the first coin is a head. Let $B$ be the event that the first coin is a tail. Let $C$ be the event that the second coin is a head.

(a) What is the sample space for this experiment?
(b) List the outcomes in $B^c$.
(c) List the outcomes in $A \cup C$.
(d) List the outcomes in $A \cap C$.
(e) List the outcomes in $A \cup B \cup C$.
(f) Are the events $A$ and $B$ disjoint?
(g) Are the events $A$ and $C$ disjoint?

2. Suppose $A$, $B$, and $C$ are events. Write expressions for the following events in terms of $A$, $B$, and $C$ (for example, the event that $A$ and $B$ both occur would be written as $A \cap B$):

(a) The event $B$ does not occur.
(b) The event $A$ occurs, but the events $B$ and $C$ do not.
(c) At least one of the three events occurs.
(d) All three of the events occur.
(e) At most two of the three events occur.

3. Suppose $\Omega = \{1, 2, 3\}$. Suppose $P(\{1, 2\}) = 0.5$ and $P(\{1, 3\}) = 0.9$. Calculate $P(\{1\})$, $P(\{2\})$, and $P(\{3\})$.

4. Suppose $A$, $B$, and $C$ are events. Suppose $P(A) = 0.2$, $P(B) = 0.4$, and $P(C) = 0.5$. Suppose also that $A$ and $B$ are disjoint.

(a) What is $P(A \cup B)$?
(b) Is it possible that $A$, $B$, and $C$ are disjoint? Explain your answer.

5. Consider the experiment of picking a number from 1 to 10 at random, which means the sample space is $\Omega = \{1, \ldots, 10\}$ and $P(\{i\}) = 1/10$ for $i = 1, \ldots, 10$.

(a) Give an example of two events $A$ and $B$ such that $P(A \cup B) = P(A) + P(B)$.
(b) Give an example of two events $C$ and $D$ such that $P(C \cup D) = P(C)$.
(c) Give an example of two events $E$ and $F$ such that $P(E \cap F) = P(E)$.

6. Suppose the sample space is the set $\mathbb{N}$ of all positive integers. Is it possible that $P(\{n\})$ is the same for all positive integers $n$? Explain your answer.
7. In a group of high school students, 20 percent play soccer, 25 percent play baseball, and 10 percent play both soccer and baseball. If a student is selected at random from the group, what is the probability that the student plays either soccer or baseball?

8. Suppose that 38 percent of California residents have visited New York, 34 percent have visited Florida, and 18 percent have visited both New York and Florida. What is the probability that a randomly chosen California resident has visited neither New York nor Florida?

9. Suppose that 38 percent of students at a university major in the sciences, and 29 percent of students at the university have a GPA of over 3.50. Suppose that 8 percent of students at the university are both science majors and have a GPA of over 3.50. If you choose a student from the university at random, what is the probability that the student is not a science major and does not have a GPA over 3.50?

10. Suppose that 53 percent of applicants to a Bioinformatics graduate program have a degree in Biology, 45 percent have a degree in Mathematics, and 16 percent do not have a degree in either subject.

   (a) What percentage of applicants have degrees in both Biology and Mathematics?
   (b) What percentage of applicants have a degree in Biology but not Mathematics?

11. Suppose $A$ and $B$ are events.

   (a) Show that $P(A \cap B^c) = P(A) - P(A \cap B)$.
   (b) Show that if $B \subset A$, then $P(A \cap B^c) = P(A) - P(B)$.

12. Suppose $A$ and $B$ are events. Show that $P(A \cap B) \geq P(A) + P(B) - 1$.

13. Suppose $A$ and $B$ are events. Show that

   $$P(A \cap B) - P(A)P(B) = P(A^c \cap B^c) - P(A^c)P(B^c).$$

**Selected Answers:**

3) $P(\{1\}) = 0.4$, $P(\{2\}) = 0.1$, $P(\{3\}) = 0.5$

7) 0.35

8) 0.46