Problem Set #3

(due Wednesday, October 20, in class)

1. Suppose $0 \leq r < s$ and Z is a bounded $\mathcal{F}_r$-measurable random variable. Define the predictable process $(X_t)_{t\geq 0}$ by

$$X_t(\omega) = Z(\omega)\mathbf{1}_{(r,s]}(t).$$

Suppose M is a continuous L^2-martingale. Show that

$$\int X \, dM = Z(M_s - M_r) \quad \text{a.s.}$$

2. Suppose M is a continuous local martingale. Suppose X and Y are in $\Lambda(\mathcal{P}, M)$. Show that for all $t \geq 0$ and all real numbers a and b,

$$\int_0^t (aX + bY) \, dM = a \int_0^t X \, dM + b \int_0^t Y \, dM \quad \text{a.s.}$$

Note: This was observed in class when X and Y are in $\mathcal{E}$ and M is a continuous L^2-martingale. The problem is to extend the result to the case when X and Y in $\Lambda(\mathcal{P}, M)$ and M is a continuous local martingale.

3. (Chung-Williams, Exercise 9, p. 55) Suppose M is an L^2-martingale, and suppose $X \in \mathcal{L}_2$. Let $Y_t = \int_0^t X \, dM$ for all $t \geq 0$. Suppose T is a finite stopping time. Show that

$$Y_T = \int \mathbf{1}_{[0,T]} X \, dM.$$

Note: We stated this result in class when $X \in \Lambda^2(\mathcal{P}, M)$ and T is bounded. Here you will show that when $X \in \mathcal{L}^2$, the condition that T is bounded can be weakened. You may use the result we stated in class if you wish.