**Homework #4**

(due Wednesday, November 5, in class)

1. Suppose $M$ is a continuous local martingale, $X \in L^2_{loc}(M)$, and $Y \in L^2_{loc}(M)$. Let $a, b \in \mathbb{R}$. Show that

$$(aX + bY) \cdot M = a(X \cdot M) + b(Y \cdot M).$$

Note: In class, we stated this result when $M$ is a bounded continuous martingale and $X$ and $Y$ are in $\mathcal{E}$. You may use the result in that case without proof.

2. Suppose $M$ is a continuous local martingale, and $X$ is a predictable process with $\|X\|_M < \infty$, where

$$\|X\|_M^2 = E\left[ \int_0^\infty X_s^2 \, d\langle M \rangle_s \right].$$

Show that $X \cdot M$ is a martingale in $\mathcal{M}^2$ and $\|X \cdot M\|_2 = \|X\|_M$.

Note: In class, we proved this result when $M$ is a bounded continuous martingale. The problem is to extend the result to the case when $M$ is a continuous local martingale.

3. Let $K \in \mathbb{R}$, and suppose $(X^n)_{n=1}^\infty$ is a sequence of predictable processes such that for all $t \geq 0$ and $\omega \in \Omega$, we have $|X^n_t(\omega)| \leq K$ for all $n$ and $\lim_{n \to \infty} X^n_t(\omega) = X_t(\omega)$. Show that for every continuous local martingale $M$ and each fixed $t > 0$, we have

$$\int_0^t X^n_s \, dM_s \to_p \int_0^t X_s \, dM_s,$$

where $\to_p$ denotes convergence in probability as $n \to \infty$.

Note: Here the notation $\int_0^t X_s \, dM_s$ has the same meaning as $(X \cdot M)_t$. 