

A gradient Gibbs measure is the projection to the gradient variables $\eta_b = \phi_y - \phi_x$ of the Gibbs measure of the form

$$P(d\phi) = Z^{-1} \exp\left\{-\beta \sum_{\langle x,y \rangle} V(\phi_y - \phi_x)\right\} d\phi,$$

where V is a potential, β is the inverse temperature and $d\phi$ is the product Lebesgue measure. The simplest example is the (lattice) Gaussian free field $V(\eta) = \frac{1}{2}\kappa\eta^2$. A well known result of Funaki and Spohn asserts that, for any uniformly-convex V , the possible infinite-volume measures of this type are characterized by the *tilt*, which is a vector $u \in \mathbb{R}^d$ such that $E(\eta_b) = u \cdot b$ for any (oriented) edge b . I will discuss a simple example for which this result fails once V is sufficiently non-convex thus showing that the conditions of Funaki-Spohn's theory are generally optimal. The underlying mechanism is an order-disorder phase transition known, e.g., from the context of the q -state Potts model with sufficiently large q . Based on joint work with Roman Kotecký.