

Additive functionals on multiway search trees

We derive asymptotics of moments and limiting distributions, under the random permutation model on m -ary search trees on n keys, of functionals that satisfy recurrence relations of a simple additive form. Many important functionals including the space requirement, internal path length, and the so-called shape functional fall under this framework.

The limiting behavior of these functionals exhibit intriguing phase changes. For suitably “small” input (or *toll*) sequences we have asymptotic normality if $m \leq 26$ and typically periodic behavior otherwise. For “moderate” toll sequences we have convergence to non-normal distributions if $m \leq m_0$ (where $m_0 \geq 26$) and typically periodic behavior otherwise. For “large” toll sequences we have convergence to non-normal distributions for all values of m .

Recent research greatly sharpens the understanding of the periodic cases. For example, Chauvin and Pouyanne have shown that for $m \geq 27$ fixed, the space requirement equals

$$\mu(n+1) + 2\Re[n^{\lambda_2}\Lambda] + \epsilon_n n^{\Re\lambda_2},$$

where $\mu \in \mathbb{R}$ and $\lambda_2 \in \mathbb{C}$ are certain constants, Λ is a complex-valued random variable, and $\epsilon_n \rightarrow 0$ a.s. and in L^2 as $n \rightarrow \infty$. Using the elementary but powerful “contraction method,” we identify the distribution of Λ .

As time and research progress permits, we will show how the periodic-case result can be extended rather generally to other additive functionals under the random permutation model, and beyond those, to generalized Pólya urn schemes and multi-type branching processes.

(This is joint work with Jim Fill.)