Review of Probability Distributions

In this course, we have introduced six different probability distributions, three for discrete random variables and three for continuous random variables. Below is a brief review of these distributions.

- A random variable X has the geometric distribution with parameter p if

$$P(X = k) = (1-p)^{k-1}p, \qquad k = 1, 2, \ldots.$$

  This is the distribution of the number of trials required to get a success if each trial is independently successful with probability p.

- A random variable X has the binomial distribution with parameters n and p if

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \ldots, n.$$

  This is the distribution of the number of successes in n trials if each trial is independently successful with probability p.

- A random variable X has the Poisson distribution with parameter λ if

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \qquad k = 0, 1, 2, \ldots.$$

  This is the distribution of the number of events that should occur during a time interval, if we expect λ occurrences on average and if events occur at a constant rate.

- A random variable X has the uniform distribution on $[a, b]$ if it has density

$$f(x) = \begin{cases} 1/(b-a) & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

  Informally, this means that X is equally likely to be anywhere between a and b.

- A random variable X has the exponential distribution with parameter λ if it has density

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

  This is the distribution of the time to wait for the next event to occur, if events occur at the constant rate λ per unit time.

- A random variable X has the normal distribution with mean μ and standard deviation σ if it has density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty.$$

  This is a symmetric, bell-shaped distribution which arises frequently in practice because of the Central Limit Theorem.