Math 10c - Practice Final

Disclaimer: There may be problems on the midterm that do not appear on this practice midterm. Conversely, there may be problems on this practice midterm that are not on the midterm. It is highly recommended that you complete the problems suggested by Professor Shopple before taking this practice midterm.

1) (8.7, 17) After measuring the duration of many telephone calls, the telephone company found their data was well-approximated by the density function $p(x) = 0.4e^{-0.4x}$ where x is the duration of a call, in minutes.

(a) What percentage of calls last between 2 and 3 minutes?

(b) What percentage of calls last 2 minutes or less?

(c) Find the cumulative distribution function.

2) (8.8, 5) Suppose that x measures the time (in hours) it takes for a student to complete an exam. All students are done within two hours and the density function for x is $p(x) = x^3/4$ if $0 < x < 2$ and 0 otherwise.

(a) What proportion of students take between 1.5 and 2.0 hours to finish the exam?

(b) What is the mean time for students to complete the exam?

(c) Compute the median of this distribution.

3) (9.1) Determine if the series is a geometric series. If it is, find the sum.

$$3 - \tfrac{3}{2} + \tfrac{3}{4} - \tfrac{3}{8} + \tfrac{3}{16} - \ldots$$

4) (10.1) Find the Taylor Series approximation of degree 4 for $f(x) = \cos x$ for x near $a = 0$.

5) (12.1) Describe the set of points whose distance from the z-axis is 3.

6) (12.2) Sketch the graph of the surface and briefly describe it in words.

(a) $x = -2$ **(b)** $x^2 + z^2 = 1$

7) (12.3) Sketch a contour diagram for the function. Label the contours $z = -1$, $z = 0$, $z = 1$, and $z = 2$.

$$z = f(x, y) = 2x^2 + y$$

8) (12.4) Find the equation of the linear function $z = c + mx + ny$ whose graph contains the points $(0, 0, 0)$, $(0, 1, 3)$, and $(-1, -2, 0)$.

9) (13.1) Find a vector with length 3 that points in the same direction as $2\vec{i} - 3\vec{j} + \vec{k}$.

10) (13.2) Shortly after takeoff, a plane is climbing northwest through still air at an airspeed of 250km/hr and rising at a rate of 200 m/min. Resolve its velocity vector into components. The x-axis points east, the y-axis points north, and the z-axis points up.

11) (13.3) Give a vector that is parallel, but not equal to $\vec{v} = 3\vec{i} - 7\vec{j}$. Give two distinct vectors that are perpendicular to $\vec{v}$.

12) (13.4) Let $P = (1, 0, 1)$, $Q = (-1, 1, 2)$, and $R = (2, 2, 3)$. Find the equation of the plane that contains P, Q, and R.

13) (14.1) The grade of a student $G = G(s, t)$, on a scale of 0 - 100, is a function of the number of hours studied s and the number of hours spent watching television t. Do you expect G_s to be positive or negative? Do you expect G_t to be positive or negative. Briefly explain your solution.

14) (14.2) Find the partial derivatives.

(a) f_x and f_y if $f(x, y) = 2\sin x \cos 3y + xy$

(b) z_x if $z = 7x^2y + 5xy - y$

15) (14.3) Find the equation of the tangent plane at the given point.

$$z = x^2 + 3y^2 \text{ at the point } (2, 1, 7)$$

16) (14.4, 21) Find the gradient of $f(x, y) = \sqrt{x^2 + y^2}$. Assume the variables are restricted to a domain on which the function is defined.

17) (14.4) Find the directional derivative $f_{\vec{u}}(1, 3)$ for the function $f(x, y) = x^2 + y$ with $\vec{u} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$.

18) (14.6, 8) Find $\partial z/\partial u$ and $\partial z/\partial v$. The variables are restricted to domains on which the functions are defined.

$$z = (x+y)e^y,\ x = \ln u,\ y = v$$

19) (14.7) Find all four second-order partial derivatives. Assume the variables are restricted to a domain on which the function is defined.

$$f(x,y) = x + (x+2y)^2 - y$$

20) (14.7) Find the quadratic Taylor polynomial about $(0,0)$ for the function $e^x + e^{2y}$.

21) (14.8, 4) Find the critical points and classify them as local maxima, local minima, saddle points, or none of these.

$$f(x,y) = x^3 + y^2 - 3x^2 + 10y + 6$$

22) (15.2, 10) Find the global maximum and minimum of the function on $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and say whether it occurs on the boundary of the square.

$$z = x^2 - y^2$$

23) (15.2, 5) Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint, if such values exist.

$$f(x,y) = x^2 + y^2, \qquad 4x - 2y = 15$$