Math 10C (Section C) Fall 2009 Midterm 2 Study Guide

A calculator is permitted during the exam, but it is not necessary to complete the problems or to get full credit (provided you can do arithmetic from prerequisite courses). One page (8.5″ by 11″) of handwritten notes (on one side) is allowed.

A summary of the topics that you should understand are listed below. The bullet points • indicate some of the main concepts from the section in addition to the title of the section. In parentheses are exercises from that textbook section that would be good for extra practice. Some problems on the exam may not be exactly like the problems listed here.

Chapter 12. You should understand this chapter (3-space, contour diagrams, cross sections, various equations of surfaces) as it applies to Chapters 13 & 14.

13.1. Displacement Vectors (1–16)

- Component vectors $\vec{i}$, $\vec{j}$, $\vec{k}$
- Resolve a vector $\vec{v}$ into components $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$
- Magnitude of a vector
- Vector from point (a, b, c) to point (x, y, z) is $(x-a)\vec{i} + (y-b)\vec{j} + (z-c)\vec{k}$

13.2. Vectors in General (6–10, 11–15, 21)

- Velocity vectors
- Properties of addition and scalar multiplication

13.3. The Dot Product (1–13, 14–20, 22–24)

- Definition
  - Geometric: $\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$
  - Algebraic: $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
- Properties: commutative, distributive (p. 666)
- Perpendicular (orthogonal) if and only if $\vec{v} \cdot \vec{w} = 0$
- Equation of the plane using a point and normal vector

13.4. The Cross Product (1–14, 15–19)

- Definition
  - Geometric: $\vec{v} \times \vec{w} = (\|\vec{v}\| \, \|\vec{w}\| \sin\theta)\, \vec{n} =$ (Area of parallelogram with edges $\vec{v}$ and $\vec{w}$) $\vec{n}$
  - Algebraic: $\vec{v} \times \vec{w} = (v_2 w_3 - v_3 w_2)\vec{i} + (v_3 w_1 - v_1 w_3)\vec{j} + (v_1 w_2 - v_2 w_1)\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$
- $\vec{v} \times \vec{w}$ is perpendicular to $\vec{v}$ and $\vec{w}$
- Right-hand rule

Review Chapter 13. Exercises 1–27

14.1. The Partial Derivative (1–9, 15–20)

- $f_x(x, y)$ is the rate of change of f when we change x and keep y fixed
- $f_y(x, y)$ is the rate of change of f when we change y and keep x fixed
- Estimating partial derivatives from a contour diagram

14.2. Computing Partial Derivatives Algebraically (1–35, 37–42)

14.3. Local Linearity and the Differential (1–16, 18–20)

- Tangent plane to the Surface $z = f(x, y)$ at a point (a, b)
- Local linearization
- Differential

14.4. Gradients and the Directional Derivative (1–38)

- Geometric properties (direction & magnitude) of the gradient vector
- Compute directional derivative $f_{\vec{u}}(a, b)$ given $f(x, y)$, $\vec{u}$ and a point (a, b)

14.6. The Chain Rule (1–14, 21–23)

- Understand how to draw a diagram to figure out how to compute derivatives of functions of two variables

14.7. Second-Order Partial Derivatives (1–28)

- Degree 1 & 2 Taylor polynomial approximating $f(x, y)$ for (x, y) near $(0, 0)$.