Math 20B (Section B) Fall 2009 Final Exam Study Guide

No calculators are allowed during the exam. One page (8.5″ by 11″) of handwritten notes (on both sides) is allowed.

The topics that will potentially be on the exam are listed below. The bullet points • indicate some of the main ideas in the section in addition to the title of the section. In parentheses are exercises from that textbook section that would be good to practice. Some problems on the exam may not be exactly like the problems listed here. The preliminary questions for each section are also good questions to test your understanding of the material as you prepare.

5.2. Definite Integral (50–62, 71–75)

- Graphical interpretation, linearity, additivity for adjacent intervals, reversing the limits of integration

5.3. Fundamental Theorem of Calculus, Part I. (1–46)

5.4. Fundamental Theorem of Calculus, Part II. (1–22, 27–36)

- With chain rule

5.5. Net or Total Change as the Integral of a Rate (1–8)

5.6. Substitution Method (1–28, 33–70, 78–91)

- Change of variables of definite integrals

6.1. Area Between Two Curves (1–12, 15–45)

- Integrating along x-axis.
- Integrating along y-axis.
- Finding points of intersection to determine limits of integration
- Splitting integrals into two parts over a region

6.2. Setting Up Integrals: Volume, Linear Density, Average Value (1–15, 20–26, 52–56)

- Cross sections perpendicular to x-axis, integrating in x-direction
- Cross sections perpendicular to y-axis, integrating in y-direction

6.3. Volumes of Revolution (1–48)

- Revolving about x-axis or a horizontal line
- Revolving about y-axis or a vertical line
- Finding points of intersection to determine limits of integration

11.3. Polar Coordinates (1–22)

- Converting from rectangular/polar to polar/rectangular coordinates
- Converting equations from rectangular/polar to polar/rectangular
- Non-uniqueness of polar coordinates

11.4. Area in Polar Coordinates (1–18)

- Setting up and evaluating integrals using the formula
- Finding correct limits of integration

S.1. Complex Numbers (1–4)

- Polar and rectangular form
- de Moivre's Theorem
- Complex n^{th} roots

7.1. Skip Numerical Integration

7.2. Integration by Parts (7–50)

7.3. Trigonometric Integrals (1–54)

- $\int \sin^m x\,dx$
- $\int \cos^m x\,dx$
- $\int \sin^m \cos^n x\,dx$
- $\int \tan^m x \sec^n x\,dx$

Supplement 2. Complex Exponentials

- Just used to solve trigonometric integrals

Supplement 3. Integration of Functions which Take Complex Values (1–9)

- Using complex exponentials as a shortcut to integrating products of sines, cosines and exponentials.
- $\int \sin(mx)\cos(nx)\,dx$ or similar
- $\int e^{mx}\sin(nx)\,dx$
- $\int e^{mx}\cos(nx)\,dx$

7.5. Skip Integrals of Hyperbolic Functions

Supplement 4. The Fundamental Theorem of Algebra

- This section is just background for doing the problems in Supp 5 and 7.6
- Understand that we can factor polynomials with real coefficients into:
  - linear and irreducible quadratic factors with real coefficients
  - linear factors with complex coefficients

Supplement 5. Partial Fraction Expansions (1–4)

7.6. The Method of Partial Fractions (5–46)

- Long division of rational functions
- Find partial fraction expansion when denominator has:
  - all linear factors like $(x-a)$
  - some repeated linear factors like $(x-a)^2$
  - irreducible quadratic factors like (x^2+4)
- Integrate the partial fraction expansion into rational functions, logarithms of polynomials and arctangents of linear or quadratic functions

7.7. Improper Integrals (5–46, 65–73)

- Recognize when an integral is improper:
  - Infinite limit of integration
  - Integrand with infinite discontinuity at a limit of integration
- Writing an improper integral as the limit of a proper integral:
  - $\int_a^\infty f(x)\,dx = \lim_{R\to\infty}\int_a^R f(x)\,dx$
  - $\int_a^b f(x)\,dx = \lim_{R\to a^+}\int_R^b f(x)\,dx$ when $f(x)$ has an infinite discontinuity at $x=a$
- Evaluate improper integrals by taking a limit (like above)
- Comparison test for improper integrals to determine if an integral converges or diverges (65–73)

Chapter 10. Sequences and Infinite Series

- The main ideas from sections 1–5 are:
  - Understand the difference between a sequence and a series
  - Determine if a sequence or series converges or diverges using the proper test or method
  - Determine the limit of a convergent sequence or series if it's possible
- Section 6, find values at which a power series converges
- Section 7, find Taylor series of a function directly or substituting into a known Taylor series.

10.1. Sequences (11–26, 43–62)

- Find the limit of a sequence
  - Theorem 1, then L'Hôpital's Rule
  - Use algebra to manipulate expressions to get things like $1/n$ or $1/n^2$ that go to zero. For example, $\frac{n}{\sqrt{n^2+n}} = \frac{1}{\sqrt{1+1/n}} \to \frac{1}{\sqrt{1+0}} = 1$ as $n\to\infty$
  - Theorem 2 & 4, Limit Laws
- Geometric sequence r^n converges when $0<r<1$
- Squeeze Theorem to show convergence
- Bounded Monotonic Sequences Converge, only needed to understand why Integral Test and Comparison Test (in 10.3) work

10.2. Summing an Infinite Series (1–2, 13–16, 18–34)

- We define an infinite series as a limit of partial sums:
  - $S_N = \sum_{n=1}^{N} a_n$
  - $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} \sum_{n=1}^{N} a_n = \lim_{N\to\infty} S_N$
- Geometric series
  - Formulas for finite and infinite geometric series
  - Infinite geometric series converge for $|r| < 1$.
- Divergence Test

10.3. Convergence of Series with Positive Terms (1–32)

- Integral Test
  - Convergence of p-Series
- Comparison Test
- Skip Limit Comparison Test

10.4. Absolute and Conditional Convergence (1–12, 19–26)

- Alternating Series Test

10.5. The Ratio and Root Test (1–24, 26–31, 34–39, 41–52)

10.6. Power Series (7–26)

- Understand what a power series is
- Find radius of convergence
- Find values of x where the power series converges. i.e. within the radius and you have to check the endpoints individually.

10.7. Taylor Series (1–40)

- Understand the definition using $a_n = \frac{f^{(n)}(c)}{n!}$
- Know how to find Taylor series for some functions by manipulation and substituting into known Taylor series.

9.1. Differential Equations (1–36)

- Separation of variables to solve some equations
- General solutions
- Initial value problems

9.2. Newton's Law of Cooling (1–9)

- The meaning of y, k and b in the differential equation $y' = -k(y - b)$
- Use information (from a word problem) and the general solution $y = b + Ce^{-kt}$ to find a particular solution, i.e. find C, b and k.

9.4. Logistic Equation (1–5)

- Understand the differential equation and its solutions.
- Any problem on the final exam will be similar to the homework problems