I) Write down a formula for the solution to the following mixed boundary value problem for the Laplace equation on the rectangle $\mathcal{R} = (0, a) \times (0, b) \subseteq \mathbb{R}^2$:

$$\Delta u = 0 \text{ in } \mathcal{R}, \quad \text{and} \quad \eta \cdot \nabla u |_{x=0} = g_1, \quad \eta \cdot \nabla u |_{y=b} = g_2, \quad u |_{x=a} = g_3, \quad \eta \cdot \nabla u |_{y=0} = g_4.$$ 
Here $\eta$ denotes the outward unit normal along $\partial \mathcal{R}$.

II) Use separation of variables and Fourier series to write down a formula for the solution to the Neumann problem:

$$\Delta u = 0 \text{ in } D, \quad (\eta \cdot \nabla u) |_{\partial D} = g.$$ 
Here $\eta$ is the outward unit normal to the disk $D = \{ x^2 + y^2 < R^2 \}$.

III) Use the formula from part II) to show that if $\Delta u = 0$ in $D$ and $(\eta \cdot \nabla u) |_{\partial D} = g$, then $\int_0^{2\pi} g(\theta) d\theta = 0$.

IV) Generalize problem III) to show that if:

$$\Delta u = 0 \text{ in } \Omega, \quad (\eta \cdot \nabla u) |_{\partial \Omega} = g,$$
where $\Omega \subseteq \mathbb{R}^n$ (n=2,3) is a bounded domain with smooth boundary, then one must have:

$$\int_{\partial \Omega} g \, dS = 0,$$
where $dS$ denotes arc length with $n = 2$ and surface area when $n = 3$. Can you think of a physical reason why this makes sense?