

MATH 231A: LECTURE 12 REFERENCES

1. DIFFERENCE QUOTIENTS AND HIGHER REGULARITY

- Difference quotients, and the difference quotient lemma are discussed on Section 5.8.2 of Evans.
- There is also a treatment of higher regularity of weak solutions to elliptic equations in Section 6.3 of Evans. The method there is to employ difference quotients as well, although the regularity theory is provided purely in terms of Sobolev spaces.

We have chosen to approach regularity theory in terms of Schauder estimates because it allows one to come very close (to within “an ϵ ”) of proving regularity for non-linear problems assuming boundedness of a “critical norm” (either the solution in L^∞ , or its gradient in L^∞ depending on the context). Furthermore, Schauder estimates motivate the more precise DeGiorgi-Nash-Moser theory to be discussed next quarter.

- A deeper discussion of difference quotients and the various notions of differentiability for Sobolev functions is given in Chapter 7 of E. Stein’s classic [1].

REFERENCES

- [1] Stein, Elias M. **Singular integrals and differentiability properties of functions.** Princeton Mathematical Series, No. 30 Princeton University Press, Princeton, N.J. 1970 xiv+290 pp.